M1 Maths

# A3-6 Domain and Range

- domain and range

Summary Learn Solve Revise Answers

## Summary

In a relation, the domain is the set of values of the independent variable and the range is the set of values of the dependent variable.

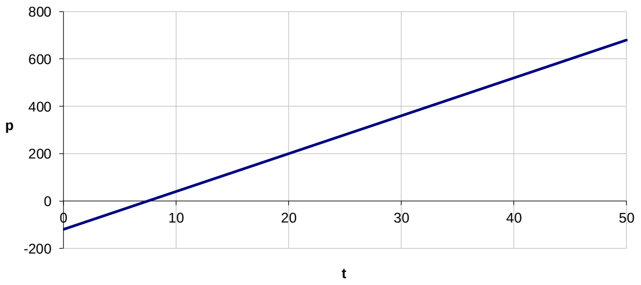
In a table, graph or set of ordered pairs, the domain and range are generally obvious. In a formula, the domain might need to be specified. It can be specified by indicating the type of numbers (e.g. integers) and the lowest and highest numbers (e.g. from −5 to 10).

If it isn't specified for a formula, then it is assumed to be the maximal domain, i.e. all the values for the independent variable for which there would be a value for the dependent variable.

## Learn

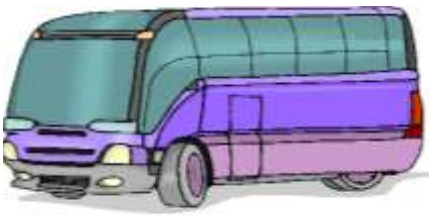
### Domain

Consider the relation $p = 16t - 120$. At first sight, this might be graphed like this.




Then consider what the relation is about. It is about the profit Richard makes with his bus.

Richard has a 14-seater mini-bus which he uses to take tourists from town to the nearby Prince's Canyon. It costs him $120 to make one trip and he charges his passengers $16 each.



The relation between profit and number of tourists he takes is thus $p = 16t - 120$, where $p$ is the profit and $t$ is the number of tourists.

Clearly, the relation does not apply to all real number values of $t$. For instance, $t$ should not exceed 14 as there are only 14 seats. Also, $t$ cannot be negative. So $t$ has to lie between 0 and 14 inclusive. Furthermore, $t$ cannot be a fraction. He cannot take a quarter of a passenger. It would make a mess of the seat.

$t$ can only be a whole number between 0 and 14 inclusive.

We can write this more concisely as $t \in W, 0 \le t \le 14$

This is the **domain** of the relation.

**The domain of a relation is the set of values for the independent variable to which the relation applies.**

The relation doesn't apply to –1 passenger, nor to 3½ passengers, but it does apply to 7 passengers.

Remember that W means the set of whole numbers. [Revisit Module N2-1, Number Sets, if you can't remember the meaning of the sets N, W, Z and R.] The items in a set are called elements of the set. The elements of the set of whole numbers are the numbers 0, 1, 2, 3 etc.

$\in$ means ***is an element of***. So $t \in W$ means *t is an element of the set of whole numbers,* or in other words *t is a whole number*.

$\le$ means less than or equal to. So $0 \le t \le 14$ means $t$ is greater than or equal to 0 and less than or equal to 14. In other words $t$ is between 0 and 14 inclusive.

## Practice

Q1 A mail order company sells plaques. They will deliver up to 5 plaques for $6 per plaque plus $10 postage and handling. Write the relation between number of plaques and cost as a formula with a domain.

Q2 What is the domain for the formula for the area of a circle $A = \frac{\pi}{4}d^2$?

State it in English and in concise mathematical language.

Q3 Express the following relation as a table and as a graph: $mass = height^3 \times 20$, where mass is in kilograms, height is in metres and $height \in N, 2 \le height \le 5$.

Q4 Present the following relation as a graph: $c = 6 + m \times 5$, where $c$ is the cost is in dollars, $m$ is the mass in kilograms and $m \in R, 2 \le m \le 10$.

## Maximal Domains

Consider the formula $y = \sqrt{x+2}$. This is an abstract formula in which we are not told the meaning of the variables and the domain isn't specified.

In such cases, we take the domain to be all the values of the independent variable which produce a real number for the dependent variable. This is called the **maximal domain**. In this case, the expression inside the radical (square root sign) must be ≥ 0 (because negative numbers do not have real square roots). In other words $x \geq -2$.

So we say the domain is $x \in R$, $x \geq -2$.

In fact, when giving a maximal domain, we sometimes assume the $x \in R$ part and just give the inequality $x \geq -2$.

Suppose $y = \frac{1}{x-2}$. This will be a real number for all $x$ except $x = 2$. It isn't a real number when $x = 2$ because then the denominator is 0 and we can't divide by 0.

So the domain is $x \in R$, $x \neq 2$.

## Practice

Q5 Write the domains for the following abstract relations:

(a) $y = \sqrt{x-4}$ (b) $y = {}^{1}\!/_{x}$ (c) $y = \frac{3}{x+1}$

## Bracket notation

Besides the inequality notation, there is another notation for expressing the domain of a relation that uses brackets.

The domain $0 < x < 8$ can be written as (0, 8)

The domain $0 \leq x \leq 8$ can be written as [0, 8]

The domain $0 < x \leq 8$ can be written as (0, 8]

We say that the domain of the relation is (0, 8].

You can probably see that a square bracket is used if the end-value is included, a round bracket (parenthesis) is used if the end-point is not included.

Note that infinity (∞) is not a number (nor is –∞), so can never be included in the domain.

## Practice

Q6 Write these domains in bracket form:
(a) $2 < x < 20$ (b) $0 \le c < 12$ (c) $-\infty < s \le 0$

Q7 Write these domains in inequality form
(a) $[0, 5]$ (b) $[-1, \infty)$ (c) $(10, 16]$

## Range

The set of allowable values for the independent variable is called the domain of the relation. The set of corresponding values for the dependent variable is called the **range** of the relation. The range is used much less frequently than the domain.

Going back to the tourist bus relation at the beginning of this unit, the relation was $p = 16t - 120$ and the domain was $t \in W$, $0 \le t \le 14$. The range is the numbers $-120$ ($p$ when $t = 0$), $-104$ ($p$ when $t = 1$), $-88$ ($p$ when $t = 2$) and so on to 104 ($p$ when $t = 14$). The range is often harder to specify concisely than the domain. In practice, this doesn't really matter because we don't often need to specify it.

In the abstract function $y = x^2$, we assume the domain to be maximal, i.e. all real numbers. But $y$ will always be 0 or positive, so the range is $y \in R$, $y \ge 0$.

## Practice

Q8 Write the range for each of these relations in bracket form:
(a) $y = x^2$, $0 \le x < 20$ (b) $y = \sqrt{x}$ (c) $y = -\sqrt{x+2}$

## Solve

Q51 Norgon Homes is a company that builds houses for a fixed rate of $800/m² plus $20 000 for services. Express this as a formula with a likely domain.

Q52 Give the maximal domain and range for the relation $x^2 + y^2 = 4$. What would the graph of this relation look like?

Q53 What would be the maximal domain of the relation $p = \frac{4}{2+\sqrt{1-s^2}}$

## Revise

### Revision Set 1

Q61 Write the relation below as a formula with a domain. Assume this is a complete relation, not a sample. Write the domain in English and in concise mathematical language.

| Time | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Voltage | 40 | 45 | 50 | 55 | 60 | 65 |

Q62 Express the following relation as a graph: $y = 2x + 3,\quad y \in \mathrm{R},\quad 4 < x < 7$

Q63 Write the domain for the abstract relation $a = \dfrac{2}{\sqrt{x}}$. What is the range?

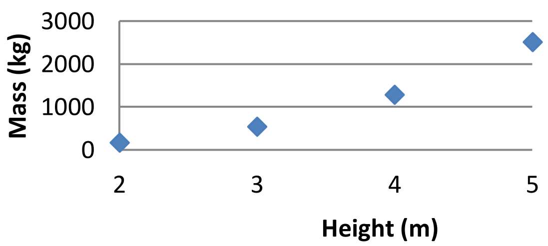
## Answers

Q1 $c = 6n + 10 \quad n \in N, 1 \le n \le 5$, where $c$ is the cost in dollars and $n$ is the number of plaques

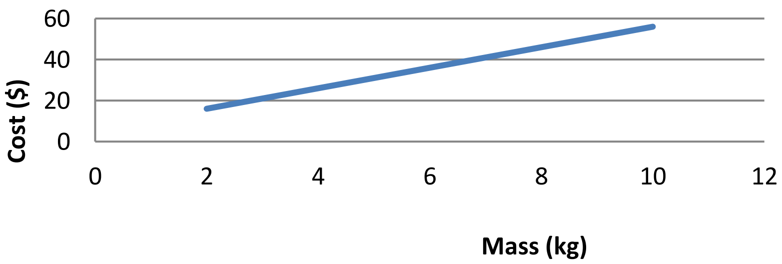
Q2 All positive real numbers $d \in R, 0 < d$

Q3

| Height (m) | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| Mass (kg) | 160 | 540 | 1280 | 2500 |




Q4




Q5 (a) $x \in R, x \ge 4$ (b) $x \in R, x \ne 0$ (c) $x \in R, x \ne -1$

Q6 (a) (2,20) (b) [0, 12) (c) $(-\infty, 0]$

Q7 (a) $0 \le x \le 5$ (b) $-1 \le x < \infty$ (c) $10 < x \le 16$

Q8 (a) [0, 400) (b) $[0, \infty)$ (c) $(-\infty, -2]$

Q51 $cost$ (\$) = $area$ ($m^2$) × 800 + 20 000; maybe $area \in \mathrm{R}, 60 \le area \le 500$

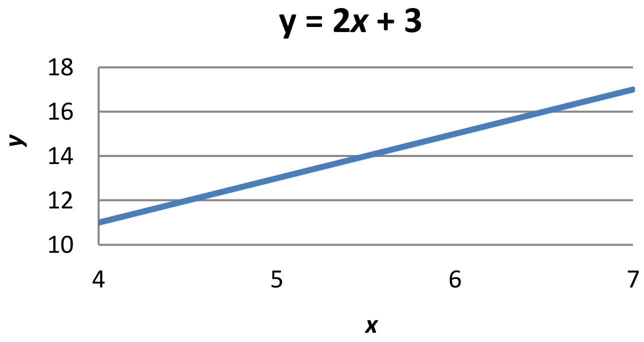
Q52 domain: $x \in \mathrm{R}, -2 \le x \le 2$; range: $y \in \mathrm{R}, -2 \le y \le 2$.

The graph would be a circle of radius 2 around the point (0, 0).

Q53 $s \in \mathrm{R}, -1 \le s \le 1$

Q71 $v = 5t + 35$ where $v$ is the voltage and $t$ is time, $t \in N$, $1 \leq t \leq 6$

Q72




Q73 $x \in R, a > 0$ $a > 0$