M1 Maths

A3-5 Solving by Graphing

- using a graphics calculator or computer graphing program
- graphing relations and solving equations using a graphics calculator

Summary   Learn   Solve   Revise   Answers

Summary

Not all equations can be solved using algebraic techniques. For instance $3^x = x^2 - \sqrt{x}$ can't. But all equations can be solved by graphing using a graphics calculator.

To do this, we go into graphing mode, then enter the expression on the left of the = sign (3^x) as Y1 and the expression on the right of the = sign ($x^2 - \sqrt{x}$) as Y2. We then draw both graphs. We then jump to the intersection point and the x-value there is the solution to the equation.

If there is more than one solution, there will be more than one intersection and we can jump from one to the next to find them all.

Students who don't have access to a graphics calculator can use an on-line graphing program like Desmos (desmos.com/calculator).

Learn

You have learnt how to solve equations algebraically.

So you can solve equations like $2(x - 3) + 4x = 7 - x$,

but you can't use the methods you've learnt to solve equations like

$$x^3 = 12, \qquad x^2 + 2x = 7, \qquad 2^x = 15 \qquad \text{or} \qquad 3^x = x^2 - \sqrt{x}$$

In later modules, you will learn methods to solve the first three of these, but there is no algebraic method for solving the last one. In fact, quite a few equations that arise when solving problems cannot be solved algebraically.

In this module you will learn a method that can solve any equation, whether or not it can be solved algebraically. The method is **graphing**.

Why then do we learn the other methods? Because solving equations by graphing is very time consuming if done by hand. It is quicker if done using a graphics calculator or computer graphing program, but it is still slower than the method you have learnt for simple equations like $2a + 5 = 11$. Also, you will probably not have a graphics calculator or computer handy every time you need to solve an equation.

In this module we won't graph by hand. We will use the calculator or computer. The instruction below or for a Casio graphics calculator. Other brand will work in quite similar ways and the instructions can be easily adapted. Students who don't have access to a graphics calculator can find the same functions on an on-line graphing program like Desmos (desmos.com/calculator).

This might be the first time you have used a graphics calculator. So we might start with an introduction. Graphics calculators can be used for all the things you can do on a scientific calculator. But they do a lot of other things too. They draw graphs of relations, they solve equations, they do statistics and they do a lot of things that you won't need until quite a bit later in your studies.

Performing arithmetic calculations on a graphics calculator

This is done fairly much the same way as on a scientific calculator. Your calculator may go automatically to computation mode when you turn it on. Some Casios require you to select the mode by pressing 1 or by pressing MENU then 1.

Note that to enter a negative number, you press the (–) key before entering the number. Don't confuse the (–) key with the subtraction key which is – without the brackets.

The calculator will probably have a DEL (delete) key and an INS (insert) key to allow you to edit what you have put in or correct mistakes. Familiarise yourself with how these work. There will also be cursor keys (arrows pointing left, right, up and down) which allow you to move around the screen. Familiarise yourself with these too.

Practice

Q1 Use your graphics calculator to perform the following calculations.

(a) 3.1×5 (b) -2×-4 (c) 3.5^2

(d) $\sqrt{20}$ (e) $2.2^{6.4}$ (f) $^2/_9 \div 3^5$

Note on Order of Operations

You already know that scientific calculators follow the order of operation conventions. This is brackets; multiplication and division together left to right; addition and subtraction together left to right.

But scientific and graphics calculators perform other operations too and it is worth being aware of the order in which they do them.

The usual order is: brackets; powers; fractions; operations like sin, cos, tan, log etc.; multiplication and division together left to right; addition and subtraction together left to right.

Graphing relations

There is a custom amongst mathematicians that, if a formula doesn't relate to a specific situation with specific variables, they use x as the abbreviation for the independent variable and y as the abbreviation for the dependent variable.

So we might use $f = 2d + 3$ for the relation between fare and distance for a taxi which charges a $3 flag fall and $2 per kilometre.

But we might use $y = 6x + 1$ for an unspecified relation in which the dependent variable is equal to 6 times the independent variable plus 1.

When you enter relations on a graphics calculator, the calculator doesn't know what the relation is about, so it uses x and y.

The procedure for graphing relations varies a bit between makes of calculator.

On a Casio press MENU then 5 (Graph). Then type in an expression to complete the formula Y1=. Note that only explicit formulae can be entered. These are formulae that have just the dependent variable on the left of the = sign, e.g. $y = 2x^2 + 6x$. Implicit formulae like $x^2 + y^2 = 4$ cannot be entered. You would need to change it to explicit form first. In this case you would need to enter $Y1 = \sqrt{4 - x^2}$ and $Y2 = -\sqrt{4 - x^2}$.

The dependent variable is always entered as Y. Y1, Y2 etc. are available to allow you to graph a number of relations at the same time. The independent variable is always entered as X. You enter it by pressing the X,θ,T key.

Once you have entered a formula, press EXE. When you have entered all the relations you wish to, press F6 (DRAW).

To go back to an earlier screen at any stage, press EXIT.

To delete a relation, go to the list, then use the cursor keys to put the cursor on the relation you wish to delete then press F2 (DEL) then F1 (YES).

Let's graph $s = k^3 - 3k$.

> In the main menu Press 5 (GRAPH)
> Press X,θ,T, ^, 3, −, 3, X,θ,T
> Press EXE
> Press F6 (DRAW)

When you graph a relation, you will generally need to set the View Window. This is the range of x-values and y-values that are shown on the graph. While the graph is showing, press F3 (V Window), then set the x-values for the left and right sides of the

graph and the y-values for the top and bottom. After setting each value, press EXE. If you want to move onto the next value without changing the present one, press the down cursor key.

It may take a bit of trial and error to get the best window for the function you are graphing.

Practice

Q2   Graph each of the following relations. Choose a view window to show the main features of the graph, then sketch what you see.

(a) $y = x^3 - 6x$ (b) $y = x^3 - 6x + 80$

(c) $h = 200t - 5t^2 + 500$ (d) $p = \sqrt{4 - x^2}$

(e) $k = -20 - 1.05^t$ (f) $f = x^x$

Solving equations graphically

To solve an equation graphically, put the expression on the left side of the = sign in Y1 and the expression on the right side of the = sign in Y2.

For instance, to solve $2^x + 5 = 5x^2 - 12x$,
put Y1 = $2^x + 5$ and Y2 = $5x^2 - 12x$.

Graph both relations.   If you use a view window with x from -2 to 6 and y from -10 to 20, you should get something like this.
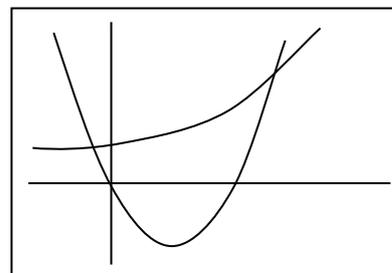
One curve shows the values of $2^x + 5$ for different values of x, the other curve show the values of $5x^2 - 12x$ for the values of x. Where $2^x + 5 = 5x^2 - 12x$ the graphs will be at the same height, i.e. they will intersect. So the x-values of the intersections are the solutions of the equation.

To get the coordinates of the intersection, while the graph is on the screen, press F5 (G Solv), then F5 (ISCT (intersection)). The calculator will move the cursor along one of the functions to the intersection. It will stop there and the x- and y-coordinates will be displayed. The x-coordinate is the solution to the equation.

As there is more than one intersection, then there is more than one solution. To find the other, press the right arrow cursor key and the cursor will move to the next intersection. Read off the x-value there.

Practice

Q3 Solve the following equations graphically.

   (a) $480 = 80t + 100$

   (b) $t^3 + 2t = 60$

   (c) $\sqrt{c+12} = 12 - c$

   (d) $5s^2 - 40 = \dfrac{2}{s} + 12$

   (e) $2h^2 + 7h = 12h + 5$

Q4 Find graphically the value of t when $d = 290$ if $d = 60t + 85$

Q5 Uncle Bert is driving to Ayr. His distance from Mackay is given by the formula $d = 95t + 50$. Use graphing to find out when he will arrive at Ayr, 313 km from Mackay.

Q6 Solve $2^n = n + 30$

Q7 Harriett invested \$4000 at 6.25% p.a. effective interest, compounding daily. This means that her balance will be given by $b = 4000 \times 1.0625^t$ where b is her balance in dollars and t is the time in years since she invested the money. After how many years will she have \$6000?

Q8 A cannon ball is fired upwards. Its height is given by the formula $h = 200t - 5t^2$ where h is the height in metres and t is the time in seconds since it was fired. How long after firing will it be 1800 m high?

Solve

Q51 A bacteria colony weighed 17 mg at 1 p.m. It then doubled its mass every hour, so its mass in mg t hours after 1 p.m. was 17×2^t. At what time (to the nearest minute) did it weigh 1 g?

Q52 The mass of a bacteria colony doubles every 3 hours. It weighs 80 mg at 8 a.m. The mass of a second colony increases at a steady 200 mg/h. At 8.a.m. its mass is 2.1 g. To the nearest minute, at what time will the two colonies have the same mass?

Q53 A sphere has a volume equal to $\dfrac{\pi}{6}$ times the cube of its diameter. What is the diameter of a sphere with volume 2 litres?

Q54 A lift is descending at 20 m/s. As the top of the lift passes the 38th floor, it dislodges a spanner which a workman had left in the lift well. The spanner accelerates under gravity such that the distance it has fallen t seconds after being dislodged is $5t^2$. If each floor is 3 m high, at which floor will the spanner next hit the lift?

Revision Set 1

Q61 Solve the following equations by graphing with a graphics calculator.

(a) $2.1x + 9 = 16.1$ (b) $p^2 + 3p - 12 = 5$ (c) $\sqrt{5m-1} = 0.2m + 2$

Q62 Find the side lengths of a cube that has a volume of 40 m³.

Q63 Garble borrowed $400 at 10% p.a. compound interest. So the amount he owed t years later was 400×1.1^t. When he paid it back, he had to pay $761. How long did he have the money for?

Answers

Q1	(a) 15.5	(b) 8	(c) 12.25	(d) 4.47	(e) 155.4	(f) $^4/_{69}$
Q3	(a) 4.75	(b) 3.74	(c) 7.58	(d) -3.21, -0.04, 3.24	(e) -0.76, 3.26
Q4	3.417	Q5 when $t = 2.77$ h		Q6 -30, 5.13	Q7 6.69
Q8	13.7 s, 26.3 s				

Q51   6:53 pm   Q52. 2:33 a.m. the next day   Q53 15.6 cm   Q54 11th floor

Q61   (a) 3.38   (b) −5.89, 2.89   (c) 1.20   Q72 3.42 m   Q73 6.75 years