M1 Maths

A3-4 Rearranging Formulae

- changing the subject of a formula

Summary   Learn   Solve   Revise   Answers

Summary

Suppose we have the $f = 3d + 0.4t + 2$. The dependent variable, f, is sometimes called the subject of the formula. We can re-write the formula with d or t as the subject, e.g. $d = \frac{f - 0.4t - 2}{3}$.

To do this, we use the same process as when solving an equation. We solve as if d is the unknown and the other variables are constants. We consider what operations were done to d and undo them, starting with the last one.

Learn

Changing the Subject of a Formula

Let's say the formula for working out the fare for a taxi ride is $f = d \times 2 + 3$, where f is the fare in dollars and d the distance travelled in kilometres. This means that there is a flag fall of \$3 (i.e. you pay \$3 just to get in), then you pay another \$2 for each kilometre you travel.

This formula makes it easy to work out the fare for any length of journey. We just substitute the number of kilometres for d and do some simple arithmetic. For example if we want to go 11.5 km, then d is 11.5. So $f = d \times 2 + 3$ becomes $f = 11.5 \times 2 + 3$. This works out to $f = 26$. So the cost is \$26.

Suppose we were more interested in working out the distance you could go for various amounts of money. A formula for d would be handy – one with d as the dependent variable and f as the independent variable – one that starts $d = \ldots$.

The formula would in fact be $d = (f - 3) \div 2$ or, written another way, $d = \frac{f-3}{2}$.

Just as a check, lets substitute in \$26 for f and make sure that d comes to 11.5 km.

$$d = \frac{26-3}{2} = \frac{23}{2} = 11.5$$

In the original formula, $f = d \times 2 + 3$, we say that f is the **subject** of the formula. This means that f is the dependent variable and that it is by itself on the left side of the formula. The formula is designed to make it easy to work out f.

In the new formula, $d = \frac{f-3}{2}$, we say that d is the **subject** of the formula. This means that d is the dependent variable and that it is by itself on the left side of the formula. This formula is designed to make it easy to work out d.

In changing from the original form of the formula to the second form, we say that we have **changed the subject of the formula**. Sometimes this is called **rearranging the formula**.

But how do we do this? The method is remarkably similar to the doing the same thing to both sides method we've used to solve equations.

Suppose we know $f = d \times 2 + 3$

We turn this round to $d \times 2 + 3 = f$ so that d is on the left – the way we want it to end up.

Then we perform inverse operations to undo the left side until we end up with just d.

$$d \times 2 + 3 = f$$
$$-3 \qquad -3$$
$$d \times 2 = f - 3$$
$$\div 2 \qquad \div 2$$
$$d = \frac{f-3}{2}$$

And the job's done.

Make sure that this makes sense, then try the following.

Practice

Q1 (a) Given that $f = d \times 3 + 5$, find a formula for d. This will be exactly the same process as the example above but with different numbers.

(b) Given that $f = d \times 2 + 4$, find a formula for d.

(c) Change the subject of $f = d \times 4 - 3$.

(d) Given that $m = s \times 3 + 1$, find a formula for s.

(e) Given that $r = p \div 3 + 7$, find a formula for p.

(f) Change the subject of $a = 5(t - 4)$.

(g) Change the subject of $m = \frac{v+5}{4}$.

(h) If $f = d \div 2 - 5$, find a formula for d.

(i) If $h = \frac{3a+2}{4} + 5$, find a formula for a.

(j) If $k = \frac{r}{4} + 1$, find a formula for r.

(k) If $s = \frac{3h-2}{5}$, find a formula for h.

(l) Change the subject of $t = \frac{4r+1}{5} - 2$

(m) Change the subject of $w = 3(u + 4) - 1$

(n) If $p = 6r$, find a formula for r.

(o) If $a = \frac{5}{v}$, find a formula for v. (Start by multiplying both sides by v.)

(p) If $h = \frac{12}{s} + 6$, find a formula for v.

(q) If $p = \frac{5}{t+2}$, find a formula for t.

(r) If $w = \frac{4}{3c-1}$, find a formula for c.

(s) If $h = 5r^2$, find a formula for r.

(t) *If* $a = 3\sqrt{c+4}$, find a formula for c.

Changing the Subject of a Formula with More Than One Independent Variable

You would be familiar with the formula for the area of a rectangle $A = l \times w$. This relation has two independent variables, l and w, and one dependent variable, A.

Suppose we wanted to change the subject of the formula $A = l \times w$ to get a formula for l.

The method for this is pretty much the same as the method we learnt for formulae with a single independent variable. It will look like this:

$$A = l \times w$$

Reverse it so l is on the left $\quad l \times w = A$

$$\div w \qquad \div w$$

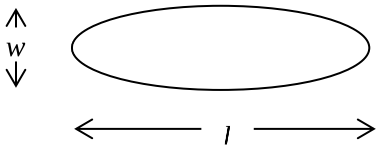
$$l = \frac{A}{w}$$

Practice

Q2 (a) The formula for the area A of a triangle of base length b and perpendicular height h is $A = b \times h \div 2$. Rearrange this formula so b is the subject.

(b) Find a formula for h in Q2(a).

(c) An ellipse is a squashed circle:



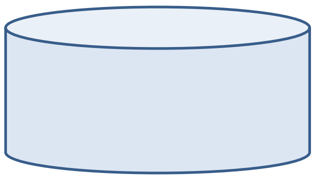
If the area is A and the length and width are l and w, then
$A = l \times w \times 0.785$.

Find a formula for l and one for w.

(d) If $r = 2nk$, find a formula for n.

(e) If $p = 3w \times (a + 4)$, find a formula for a and one for w.

(f) The volume of a cylinder of diameter d and height h is $V = 0.785d^2h$. Find a formula for h and a formula for d.



(g) When a voltage of V volts is applied across a resistor with resistance R ohms, the number of amps, I, of electric current that flow through the resistor is given by the formula $I = \frac{V}{R}$. Find a formula for the voltage needed to get a current of I amps to flow through a resistor of R ohms.

Solve

Q51 Make c the subject of $h = 3c + ac$

Q52 Make x the subject of $k = x + \frac{x}{2} + \frac{x}{4} + \frac{x}{8} + \frac{x}{16} + \frac{x}{32} + \frac{x}{64} + \frac{x}{128} + \ldots$ and so on for ever

Q53 Draw a grid from $x = -10$ to $x = 10$ and from $y = -10$ to $y = 10$. On it, plot the graphs of $y = x^2 + 1$, $y = -x^2 - 1$, $x = y^2 + 1$ and $x = -y^2 - 1$. [Don't forget that $3^2 = 9$, but that $(-3)^2 = 9$ also.] What do you notice?

Revise

Revision Set 1

Q61 (a) If $f = d \times 4 + 2$, find a formula for d.

(b) Change the subject of $w = \frac{4b-1}{7}$

(c) If $t = \frac{5}{2c+7}$, find a formula for c.

(c) The volume of a square-based pyramid of side length s and height h is $V = d^2h \div 3$. Find a formula for h.

(d) The volume of a square-based pyramid of side length s and height h is $V = d^2h \div 3$. Find a formula for d.

Answers

Q1 (a) $d = \frac{f-5}{3}$ (b) $d = \frac{f-4}{2}$ (c) $d = \frac{f+3}{4}$ (d) $s = \frac{m-1}{3}$

(e) $p = 3(r-7)$ (f) $t = \frac{a}{5} + 4$ (g) $v = 4m - 5$ (h) $d = 2(f+5)$

(i) $d = \frac{4h-22}{3}$ (j) $r = 4(k-1)$ (k) $h = \frac{5s+2}{3}$ (l) $r = \frac{5t+9}{4}$

(m) $u = \frac{w+1}{3} - 4$ (n) $r = \frac{p}{6}$ (o) $v = \frac{5}{a}$ (p) $s = \frac{12}{h-6}$

(q) $t = \frac{5}{p} - 2$ (r) $c = \frac{4+w}{3w}$ (s) $r = \pm\sqrt{\frac{h}{5}}$ (t) $c = \frac{a^2}{9} - 4$

Q2 (a) $b = \frac{2A}{h}$ (b) $h = \frac{2A}{b}$ (c) $l = \frac{A}{0.785w}$, $w = \frac{A}{0.785l}$

(d) $n = \frac{r}{2k}$ (e) $a = \frac{p}{3w} - 4$, $w = \frac{p}{3(a+4)}$ (f) $h = \frac{V}{0.785d^2}$, $d = \sqrt{\frac{V}{0.785h}}$

(g) $V = IR$

Q51 $c = \frac{h}{3+a}$ Q52. $x = \frac{k}{2}$

Q53 A picture with mirror and rotational symmetry

Q61 (a) $d = \frac{f-2}{4}$ (b) $b = \frac{7w+1}{4}$ (c) c = $\frac{5-7t}{2t}$

(d) $h = \frac{3V}{d^2}$ (e) $d = \sqrt{\frac{3V}{h}}$