M1 Maths

A3-3 Squares and Fractions

- solve equations with squares and fractions

Summary Learn Solve Revise Answers

Summary

In some equations the variable is squared or square-rooted. Squaring and square-rooting are inverse operations and each can be undone with the other.

Because a negative number squared is positive, the square root of a positive number can be positive or negative and both possibilities need to be considered when taking a square root.

Equations may also involve fractions or percentages of unknowns. These can be solved by treating say $^3/_5$ of the unknown or 60% of the unknown or as 0.6 × the unknown, then using division as the inverse operation.

Learn

Squares

We haven't as yet solved equations like $x^2 = 16$. But this is quite easy. Obviously, if $x^2 = 16$, then $x = 4$. 4 is the square root of 16.

We can look at this in terms of inverse operations. The inverse operation of squaring is taking a square root or square-rooting. So we square-root both sides: the square root of x^2 is x and the square root of 16 is 4.

[Actually, square-rooting is not a generally accepted mathematical term, but it is easier to say than 'taking the square root of', so we will use it here. The first sentence of the summary above would be awkward if we had to say it properly.]

To lay this out we could write:

$$x^2 = 16$$
$$\sqrt{\quad} \qquad \sqrt{\quad}$$
$$x = 4$$

If squaring is one of several steps, then we undo them all starting with the last one done as usual. Like this:

$$2(x+3)^2 - 41 = 9$$
$$+41 \qquad +41$$
$$2(x+3)^2 = 50$$
$$\div 2 \qquad \div 2$$
$$(x+3)^2 = 25$$
$$\surd \qquad \surd$$
$$x+3 = 5$$
$$-3 \qquad -3$$
$$x = 2$$

In the same way, squaring is the inverse of square-rooting, so we can solve $\sqrt{2x} = 3$ like this

$$\sqrt{2x} = 3$$
$$\square \qquad \square$$
$$2x = 9$$
$$\div 2 \qquad \div 2$$
$$x = 4.5$$

Practice

Q1 Solve the following equations

(a) $x^2 = 25$ (b) $x^2 = 4$

(c) $x^2 = 1$ (d) $x^2 = 5$

(e) $x^2 = 46$ (f) $(x+2)^2 = 36$

(g) $(2x)^2 = 64$ (h) $x^2 + 6 = 22$

(i) $2x^2 = 32$ (j) $5(x+1)^2 = 20$

(k) $(2x+1)^2 - 5 = 44$ (l) $\frac{(3x+4)^2}{5} = 5$

Q2 Solve these equations

(a) $\sqrt{x} = 3$ (b) $\sqrt{x} = 9$

(c) $\sqrt{x} = 4.5$ (d) $\sqrt{2x} = 4$

(e) $\sqrt{x+5} = 6$ (f) $\sqrt{2x-1} = 5$

(g) $5\sqrt{x} = 10$ (h) $2\sqrt{3x} = 12$

(i) $4\sqrt{4x-5} = 12$ (j) $-3\sqrt{2(x+4)} + 12 = 0$

What we have done isn't strictly totally correct: there is one slight complication with square-rooting.

As you know, 3 squared is 9, so 3 is the square root of 9.

But, −3 squared is also 9 (remember if you multiply two negatives, the result is positive). So –3 is also the square root of 9. So 9 has two square roots: 3 and –3

So when we solve an equation with a square in it, we will get two solutions. We should lay it out like this:

$x^2 = 16$

$\sqrt{\quad}\quad\sqrt{\quad}$

$x = 4 \quad \text{or} \quad x = -4$

If there are more operations to undo after the squaring, then we have to do them to both alternatives like this

$2(x + 3)^2 = 50$

$\div 2 \qquad \div 2$

$(x + 3)^2 = 25$

$\sqrt{\quad}\qquad\sqrt{\quad}$

$x + 3 = 5 \quad \text{or} \quad x + 3 = -5$

$-3 \quad -3 \qquad\qquad -3 \quad -3$

$x = 2 \quad \text{or} \quad x = -8$

Making x equal to 2 will make the equation true, but so will making x equal to −8. Try it.

Solving equations which have a square root by squaring only produces one solution because squaring always produces a positive number.

Practice

Q3 Solve the following equations properly:

(a) $x^2 = 36$

(b) $x^2 = 4$

(c) $x^2 = 26$

(d) $(x + 1)^2 = 25$

(e) $(3x)^2 = 81$

(f) $x^2 - 8 = 22$

(g) $5x^2 = 80$

(h) $2(x + 3)^2 = 72$

(i) $(5x + 2)^2 - 5 = 44$

(j) $\frac{(2x+2)^2}{5} = 5$

(k) $5\sqrt{3x} = 15$

(l) $2\sqrt{6(x-1)} - 17 = 1$

Note: The square root symbol is taken by mathematicians to mean just the positive square root. So $\sqrt{16}$ means the positive square root of 16, i.e. 4, not −4. The negative square root of 16 would be written $-\sqrt{16}$. To indicate both square roots, we would write $\pm\sqrt{16}$, pronounced "Plus or minus root 16."

Note also: No real number produces a negative number when squared. So negative numbers do not have real square roots. An equation that involved the square root of a negative number does not have a real solution.

Fractions

The problems you have solved in the algebra strand up to now have involved mainly whole numbers. Some equations involve fractions or percentages of unknowns. These can be solved by treating say 60% of the unknown or $^3/_5$ of the unknown as 0.6 × the unknown, then using division as the inverse operation.

Don't forget that taking say 20% off an amount means you have 80% of that amount or 0.8 times that amount. In the same way, adding 30% means you have 130% or 1.3 times the amount.

For example, suppose we have the problem: 'In a 40%-off sale, a pair of jeans is reduced to $25.80. What was the original price?' We would solve it like this:

Let the original price be p

$p \times 0.6 = 25.8$

$\div 0.6 \qquad \div 0.6$

$p = 43$

So the original price was $43.

Practice

Q4 Solve the following by writing and solving an equation

(a) If one sixth of Gino's money is $2, how much does he have?

(b) ¾ of the people at a concert were old ladies. If there were 219 old ladies there, what was the total attendance?

(c) Three fifths of the inhabitants of Slygonville are serious criminals. Another 28 are non-serious criminals. If the total number of criminals in Slygonville is 85, how many people live in Slygonville?

(d) If 20% of Hugo's money is $80, how much does Hugo have?

(e) Joe scored 12 marks on a test. This was 60%. What was the test out of?

(f) Hairy earnt some money on Tuesday. On Wednesday he earnt one third as much as on Tuesday. All up he earnt $240. How much did he earn on Tuesday?

(g) Gnome stole some money on Thursday. On Friday he stole 20% of the amount he stole on Thursday. This gave him a total haul of \$90. How much did he steal on Thursday?

(h) Peta bought a rock. She sold it for the amount she bought it for plus 10% (i.e. she made a 10% profit). If she sold it for \$71.50, how much did she buy it for?

(i) Barbie bought some sausages and sold them later for a 120% profit. If she sold then for \$44, how much did she pay for them?

(j) Pharoah bought a pyramid and sold it later at a 70% loss. If he sold it for 6 million gold hilgas, how much did he pay for it?

(k) A pair of shoes rises in price by 12% to \$78.40. What was the original price?

(l) In a 30%-off sale a concrete rhinoceros is reduced to \$27.93. What would it have cost before the sale?

(m) A shop buys a book. They then mark up the price by 40% to sell it. It doesn't sell so they reduce it by 20%. The reduced price is \$16.80. What did the shop pay for the book? [The answer is not \$14.]

(n) 40% of the insects in Carl's collection are beetles. 10 of the insects are cockroaches. If the number of beetles and cockroaches is 38, how many insects in his collection?

(o) Sandy had 17 green marbles and a jar full of red marbles. But yesterday she sold 30% of her marble collection to her brother for \$40, leaving her with 98 marbles. How many red marbles were there in the jar before she sold some?

Solve

Q51 Solve $x^2 - 15x = 0$. Because the equation contains a square, there are two solutions. Can you find them both?

Q52 The square of any real number is positive. Therefore negative numbers cannot have real numbers as square roots. But they can have imaginary numbers as square roots. Real numbers and imaginary numbers together are referred to as 'complex numbers' Find out a bit about imaginary numbers and complex numbers.

Revise

Revision Set 1

Q61 Solve these equations.

(a) $x^2 = 49$ (b) $a^2 = 20$ (c) $3(2x - 5)^2 + 4 = 79$

(d) $\sqrt{s} = 6$ (e) $\sqrt{d} = 2.9$ (f) $10\sqrt{3(h + 2)} - 32 = 58$

Q62 Solve the following by writing and solving equations.

(a) Methuselah earnt some money on Tuesday. On Wednesday he earnt three quarters as much as on Tuesday. All up he earnt $210. How much did he earn on Tuesday?

(b) In a 40%-off sale, a rubber dinosaur is reduced to $9.30. What would it have cost before the sale?

(c) Saffie had some mice. 74 of them were white and the rest were brown. But then she ate 15% of her mice, leaving herself with 357. How many brown mice did she have before she ate some?

Answers

Q1	(a) 5	(b) 2	(c) 1	(d) 2.24	(e) 6.78	(f) 4
	(g) 4	(h) 4	(i) 4	(j) 1	(k) 3	(l) $\frac{1}{3}$
Q2	(a) 9	(b) 81	(c) 20.25	(d) 1	(e) 31	(f) 13
	(g) 4	(h) 12	(i) 3.5	(j) 4		
Q3	(a) 6, −6	(b) 2, −2	(c) 5.1, −5.1	(d) 4, −6	(e) 3, −3	(f) 5.48, −5.48
	(g) 4, −4	(h) 3, −9	(i) 1, −1.8	(j) 1.5, −3.5	(k) 3	(l) 14.5
Q4	(a) $12	(b) 292	(c) 95	(d) $400	(e) 20	(f) $180
	(g) $75	(h) $65	(i) $20	(j) 20 million	(k) $70	(l) $39.90
	(m) $15	(n) 70	(o) 123			

Q51 $x = 15$, $x = 0$

Q61	(a) 7, −7	(b) 4.47, −4.47	(c) 5, 0	(d) 36	(e) 8.41	(f) 25
Q62	(a) $120	(b) $15.50	(c) 346			