M1 Maths

A3-1 Undoing the Unknown

- undo the unknown to solve equations

Summary Learn Solve Revise Answers

Summary

So far, in solving equations, we have added, subtracted, multiplied or divided both sides by the same number. But sometimes we need to add, subtract, multiply or divide by the unknown.

Learn

Look at the equation $5a + 2 = a + 18$. You haven't learnt how to solve this as yet. See if you can work out what you would have to do.

So far, when solving equations, we have added numbers to both sides, subtracted numbers from both sides, multiplied both sides by a number and divided both sides by a number. What we need to do here is to subtract the unknown the way we have previously subtracted numbers.

To solve the equation, we would start by subtracting a from both sides.

$$5a + 2 = a + 18$$
$$-a \quad -a$$
$$4a + 2 = 18$$

Then we continue as normal.

$$-2 \quad -2$$
$$4a = 16$$
$$\div 4 \quad \div 4$$
$$a = 4$$

Sometimes we will need to expand brackets and/or collect terms before we undo the unknown. For example, to solve $4(x - 2) + x = 10 + 3x$, we would proceed like this:

$$4(x - 2) + x = 10 + 3x$$
$$4x - 8 + x = 10 + 3x$$
$$5x - 8 = 10 + 3x$$
$$-3x \quad -3x$$
$$2x - 8 = 10$$

$$\begin{array}{r} +8 \quad +8 \\ 2x = 18 \\ \div 2 \quad \div 2 \\ x = 9 \end{array}$$

Practice

Q1 Solve the following equations.

(a) $5c - 12 = c + 4$ (b) $4w + 1 = w + 10$

(c) $3p + 2 + p - 4 = p + 7$ (d) $3(d + 4) = d + 20$

(e) $6f - 8 = 2f + 14$ (f) $5(t - 3) + 2 = 4t - 1$

If you have say $3x$ on the left and $5x$ on the right, then subtract the $3x$ rather than the $5x$. In general, it is easier to subtract the smaller number so you end up with a positive multiple of x.

Practice

Q2 Solve the following equations

(a) $3r + 12 = 5r$ (b) $a - 6 = 3a + 8$

(c) $3(u + 2) = 5u$ (d) $2(c - 4) = 5(c + 1) - 10$

(e) $2(s - 6) = 2 + 5(s + 2)$ (f) $6(m - 2) + m + 7 = 3(m + 5) - m$

(g) $5 + 4k = -2 + 7(k - 1)$ (h) $3(p + 3) = 2(2 + 2p) + 3(p - 7)$

If the unknown you are getting rid of is negative or subtracted, it can be better to add it instead of subtracting it. Try to do this so that you end up with a positive multiple of the unknown on one side. The best way to solve $5x = 8 - 3x$ is to start by adding $3x$ to both sides to get $8x = 8$.

Practice

Q3 Solve the following equations

(a) $3c = 12 - c$ (b) $-c = 4c - 15$

(c) $7d + 9 = 36 - 2d$ (d) $-3w = 8 - 7w$

(e) $2 - 7f = -3f + 14$ (f) $2(h - 2) = -3(h + 1)$

(g) $-6(-s + 8) = 40 - 2s$ (h) $22 - (n - 5) = 10$

(i) $4(3 - a) = -2a$ (j) $5 - 2d = 3(18 - 3d)$

In each of the equations so far, you had to add or subtract the unknown (or some multiple of the unknown). In other situations you might have to multiply by the unknown. For instance you might solve $20 \div p = 5$ like this:

$$20 \div p = 5$$
$$\times p \qquad \times p$$
$$20 = p \times 5$$
$$\div 5 \qquad \div 5$$
$$4 = p$$

Practice

Q4 Solve the following equations

(a) $6 \div a = 3$

(b) $42 \div r = 7$

(c) $30 \div s = 8$

(d) $11.3 \div f + 4 = 6$

(e) $8.1 \div h + 1.3 = 4$

(f) $\frac{15}{x} - 7 = 0$

(g) $15 \div s - 4 = 2.4$

(h) $(s + 9) \div s = 3$

(i) $(2b + 12) \div b = 14$

(j) $\frac{4.5}{x} = 1.93$

In some cases, you will need to multiply both sides by an expression which includes the unknown. To solve $\frac{3}{1-p} = 7.2$, the first step will be to multiply both sides by $(1 - p)$ like this:

$$\frac{3}{1-p} = 7.2$$
$$\times (1-p) \quad \times (1-p)$$
$$3 = 7.2\,(1-p)$$
$$3 = 7.2 - 7.2p$$
$$+ 7.2p \qquad + 7.2p$$
$$7.2p + 3 = 7.2$$
$$-3 \qquad -3$$
$$7.2p = 4.2$$
$$\div 7.2 \qquad \div 7.2$$
$$p = 0.583$$

In some cases there might be two expressions on the bottom, both containing the unknown. In such cases, you will have to multiply both sides by both expressions like this:

$$\frac{2}{h} = \frac{5}{h-1}$$

$\times h \qquad \times h$

$$2 = \frac{5h}{h-1}$$

$\times (h-1) \quad \times (h-1)$

$$2(h-1) = 5h$$

$$2h - 2 = 5h$$

$-2h \qquad -2h$

$$-2 = 3h$$

$\div 3 \qquad \div 3$

$$-\tfrac{2}{3} = h$$

Practice

Q5 Solve the following equations

(a) $\frac{2}{1-p} = 5$

(b) $\frac{12}{t+4} = 5$

(c) $\frac{10.3}{-c} = 4.3$

(d) $\frac{10}{4-3m} = 9$

(e) $\frac{w+1}{5-3w} = \frac{1}{2}$

(f) $\frac{8}{6-2f} = \frac{-3}{f-4}$

(g) $\frac{2}{h} = \frac{5}{h-1}$

(h) $-\frac{1}{1-w} = \frac{4}{1+w}$

Q6 Solve the following by writing and solving an equation.

(a) Amanda thought of a number. She subtracted 7, then multiplied by 3, then added 1. This gave her the number she started with. What number did she start with?

(b) Goliath thought of a number, multiplied it by 5 and subtracted 13. This gave him 3 more than he started with. What number did he start with?

(c) Fabbbledaggle thought of a number, multiplied it by 4, then subtracted 18. This gave her twice the number she started with. What number did she start with?

(d) Maggot thought of a number. It was 7.

(e) Ifrid thought of a number, subtracted 5, then multiplied by 5, then took off 3 times the number he started with. This gave him 10 less than he started with. What number did he start with?

(f) Joy thought of a number, added 6, then multiplied by 3. This gave her 25 more than twice the original number. What was the original number?

(g) Mr Lemmon thought of a number, multiplied it by 4 and added 5. This gave him 6 less than he would have got by adding 7 then multiplying by 3. What number did he think of?

(h) Opia thought of a number, doubled it, then subtracted 6. This gave her half the number she started with. What number did she start with?

(i) Thomas thought of a number, doubled it, then subtracted 8, then multiplied it by 3, then subtracted the number he started with. He ended up with 4 less than twice the number he started with. What number did he start with?

(j) Stiffo was dead.

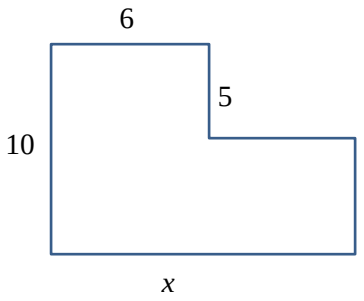
(k) Uri divided 224 by his favourite number and got 14. What was his favourite number?

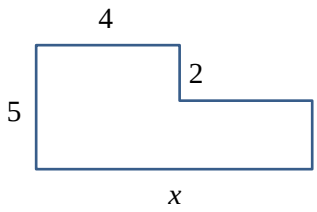
(l) The height of a triangle divided by its base is 0.37. If the height is 12 cm, what is its base?

(m) Yobs got the measles. On Monday she had a few spots. On Tuesday she had 11 more. On Wednesday she had three times as many as on Tuesday. By Thursday, all the ones she had on Tuesday had disappeared. This left her with 47 more than she had had on Monday. How many did she have on Monday?

Solve

Q51 The L-shaped room on the left has twice the area of the L-shaped room on the right. Find the value of x.





Q52 Solve $x^2 - 15x = 0$. There are two solutions. Can you find them both?

Revise

Revision Set 1

Q61 Solve the following. Show working.

(a) $4n + 5 = 25 - n$

(b) $38 - 5a = 22$

(c) $\frac{18}{x} + 11 = 0$

(d) $(2b + 32) \div b = 10$

(e) $\frac{6}{1+t} = 8$

(f) $\frac{4}{a} = \frac{11}{2a+3}$

Q62 Ginny thought of a number, added 3, then multiplied by 5, then subtracted 8. He ended up with 4 more than twice the number he started with. What number did he start with?

Q63 Jonno had enough money to buy 3 oosalem birds. He then spent $40, then lost ¾ of what he had left in a bet, then earned another $155 , then bought one oosalem bird. This left him $72 less than he would need to buy 2 more oosalem birds. How much does an oosalem bird cost?

Answers

Q1	(a) 4	(b) 3	(c) 3	(d) 4	(e) 5.5	(f) 12
Q2	(a) 6	(b) 7	(c) 3	(d) −1	(e) −8	(f) 2
	(g) $^{14}/_{3}$	(h) 6.5				
Q3	(a) 3	(b) 3	(c) 3	(d) 2	(e) −4	(f) $-^{1}/_{5}$
	(g) 11	(h) 22	(i) 6	(j) 7		
Q4	(a) 2	(b) 6	(c) 3.75	(d) 5.65	(e) 3	(f) $^{15}/_{7}$
	(g) 2.34	(h) 4.5	(i) 1	(j) 2.33		
Q5	(a) $^{3}/_{5}$	(b) $-^{8}/_{5}$	(c) −2.40	(d) $^{26}/_{27}$	(e) 7	(f) 7
	(g) $-^{2}/_{3}$	(h) $^{5}/_{3}$				
Q6	(a) 10	(b) 4	(c) 9	(d) 7	(e) 15	(f) 7
	(g) 10	(h) 4	(i) $6^{2}/_{3}$	(j) -	(k) 16	
	(l) 32.4	(m) 25				

Q51 14

Q52 $x = 15$, $x = 0$

Q61	(a) 4	(b) 3.2	(c) -1.636	(d) 4	(e) $-^{1}/_{4}$	(f) 4

Q62 –1

Q63 $96.44