

A2-3 Expanding

- expand brackets to solve equations

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Summary

When there are brackets in an equation, it is often necessary to get rid of the brackets. This process is called expanding.

If terms are added or subtracted inside a bracket and the bracket is multiplied or divided by a number outside, then to get rid of the bracket we multiply the number outside by each term inside the bracket in turn and add the results. But we take the + or – signs before each term as the sign of that term.

Learn

Another trick you have to learn is expanding. You don't do it by eating a lot.

Try to solve this equation:

$$(c + 2) \times 3 + 2c = 31$$

The unknown occurs more than once, but you can't collect terms because one of them is inside a bracket.

You have to get rid of the bracket first.

$(c + 2) \times 3$ means 3 lots of $c + 2$. This could be written as $c+2 + c+2 + c+2$. If we do this, the equation becomes

$$c + 2 + c + 2 + c + 2 + 2c = 31$$

Now we can collect terms to get

$$5c + 6 = 31$$

Which we can solve to get $c = 5$.

Practice

Q1 Solve these

(a) $(c + 3) \times 2 + 4c = 12$

(b) $(d + 5) \times 4 + d = 55$

$$(c) (w + 1) \times 3 + 2w + 4 = 27$$

$$(d) 8(2w + 5) + 3w = 59$$

You might have found that some of the last equations in that exercise got very long when you wrote them out in long form. The last one would have started

$$2w + 5 + 2w + 5 + 2w + 5 + 2w + 5 + 2w + 5 + \dots$$

and continued on for 8 lots of $2w + 5$

Now you can see why this process of getting rid of the brackets is called 'expanding'.

But there is a shorter way of getting rid of brackets which doesn't involve writing out $2w + 5$ eight times. You may have discovered it already. It is this:

$8(2w + 5) = 2w + 5 + 2w + 5 + 2w + 5 + 2w + 5 + 2w + 5 + 2w + 5 + 2w + 5 + 2w + 5$, which is 8 lots of $2w$ and 8 lots of 5 +, in other words $16w + 40$.

To expand, you just have to multiply what is outside the bracket by each term inside the bracket: $8 \times 2w = 16w$ and $8 \times 5 = 40$. Hence $16w + 40$.

$$12(s + 4) = 12 \times s + 12 \times 4 = 12s + 48. \text{ And so on.}$$

Practice

Q2 Solve these

$$(a) 2(h + 2) + 5 + 7h = 45$$

$$(b) 3v + 4(v + 3) - 2 = 45$$

$$(c) 2(m + 1) + 3(m + 3) = 41$$

$$(d) 3(n + 2) + 5n + 5(n + 4) + 6 = 175$$

$$(e) 5(h + 3) + 6 + 2h = 49$$

$$(f) 2v + 4(3v + 1) - 12 = 34$$

$$(g) 4(2m + 3) + 2(m + 6) = 84$$

$$(h) 6(n + 2) - 5n + 2(4n + 3) + 1 = 19$$

Negatives

If there are any negatives, you use what you know about multiplying negatives, i.e.

- multiplying two positive numbers produces a positive number
- multiplying a positive number and a negative number produces a negative number
- multiplying two negative numbers produces a positive number

So, for example,

$$5a + 2(a + 3) = 5a + 2a + 6 = 7a + 6$$

$$5a + 2(a - 3) = 5a + 2a - 6 = 7a - 6$$

$$5a + 2(-a + 3) = 5a - 2a + 6 = 3a + 6$$

$$5a + 2(-a - 3) = 5a - 2a - 6 = 3a - 6$$

$$5a - 2(a + 3) = 5a - 2a - 6 = 3a - 6$$

$$5a - 2(a - 3) = 5a - 2a + 6 = 3a + 6$$

$$5a - 2(-a + 3) = 5a + 2a - 6 = 7a - 6$$

$$5a - 2(-a - 3) = 5a + 2a + 6 = 7a + 6$$

Note that the plus or minus sign before a term can be taken as the sign of that term (positive or negative). So the + 2 and the - 2 before the brackets above can be read as 2 and -2; and the + 3 and - 3 inside the brackets can be read as 3 and -3. Then you just multiply the number outside the bracket by each of the numbers inside the bracket, taking note of the sign of each.

This is not an easy idea, but it is an important one. Make sure you understand how it applies to the examples above. For instance, in the last example, -2 (negative 2) is multiplied by $-a$ (negative a) to get $2a$, then -2 is multiplied by -3 to get +6.

One final case that might cause confusion is this:

$$5a - (a - 3)$$

We can think of this as meaning $5a - 1(a - 3)$, then multiply -1 by a to get $-a$, and multiply -1 by -3 to get +3, giving us $5a - a + 3$;

or we can think of the minus sign outside of the bracket as meaning '*the negative of*' or '*the opposite of*', so the negative of a is $-a$ and the negative of -3 is +3.

Expanding when there are negatives takes some getting used to. So Q3 has quite a few expansions to do. So you can get through them fairly quickly, you don't have to solve an equation - just do the expansion.

Practice

Q3 Expand the following. Collect terms where possible.

(a) $3(a + 2)$

(b) $5(x - 3)$

(c) $8(-c + 1)$

(d) $-3(s + 4)$

(e) $-4(d - 2)$

(f) $a + 3(a + 6)$

(g) $2a + 2(a - 3)$

(h) $3x - 2(x + 7)$

(i) $n - 3(n - 2)$

(j) $h - (h + 3)$

(k) $s - (s - 3)$

(l) $4w - (2w + 5)$

(m) $2a - (3 - a)$

(n) $5 + 2(-a + 3)$

(o) $2 - 3(1 - r)$

(p) $3n - (3n - 2)$

(q) $5a - 3 + (-3 - a)$

(r) $2a - 1 - 2(4 - 3a)$

(s) $-p - 2(3p + 1)$

(t) $3(d - 4) + (2d - 4)$

(u) $-4(a - 3) - (-3 - 2a)$

(v) $2(-d - 2) - (7 - 5d)$

(w) $-(-2x - 3) - (4 - x) - 15$

(x) $-8(-3a + 5) + (-a - 1)$

Q4 Now solve these equations.

(a) $2(a - 4) + a = 7$

(b) $b - 3(b + 4) = -30$

(c) $2x - (2 - 3x) = 58$

(d) $-t - (-2t - 3) = 1$

(e) $2 - 3w - (3 - 4w) + 1 = 17$

(g) $6v - 2(v - 5) = 16$

(i) $-2(3a + 1) - (1 - a) = 12$

(k) $8(a + 4) + 2a = 62$

(m) $20(t + 1) - 15t - 20 = 45$

(o) $7(4s - 2) - 25s = 13$

(f) $p + (-2 + 2p) - 5 = 23$

(h) $2(3 - 2n) - (n + 2) = 16$

(j) $6 - (2f - 5) = 0$

(l) $10(p + 3) + 13 - 6p = 87$

(n) $12(2b + 5) + 3b = 195$

(p) $25(r + 1) - 8(2r - 2) + 5 = 55$

Q5 Solve the following by writing and solving an equation.

- (a) Mazmo thought of a number, added 1, then multiplied the result by 4. He then added the number he started with. This gave him 49. What number did he start with?
- (b) Offal thought of a number, multiplied it by 6, subtracted 10, then subtracted twice the number he started with. This gave him 26. What number did he start with?
- (c) Quob had some money. Then he earned \$6. Then he made a bet and doubled his money. Then he wasted \$10 leaving himself with \$28. How much did he have to start with?
- (d) Rudolph had some money, but put it on a bet and lost it all. If he ended up with nothing, how much did he start with?
- (e) Rachelle had 3 packs of lollies, each with 4 missing, 2 packs, each with 3 extra ones in and 1 pack with the right number in. She had 132 lollies altogether. How many should there be in a pack?
- (f) Yvette thought of a number, added 7, then multiplied by 5. She then took away twice the number she started with. She ended up with 65. What number did she start with?
- (g) Zaphod thought of a number, subtracted 2, multiplied by 3, subtracted 7, then added twice the number he first thought of, then added 10, then subtracted 4 times the number he started with, then added 6. He ended up exhausted.
- (h) After he had recovered, Zaphod went through the whole procedure again and ended up with 18. What number did he start with?
- (i) Brucey bought a buzzard. The next day it had increased its weight by 20 grams. The next day it doubled its weight. The next day it doubled it again. The next day it lost 15 grams. The next day it lost the same number of grams that it had weighed when it was bought. Then it carked it. If the corpse weighed 485 grams, how much did the bird weigh when Brucey bought it?
- (j) This question was too hard and has been omitted.
- (k) Ada is 9 years older than Di. If you add twice Ada's age to 3 times Di's age then subtract 68 you will get 100. How old is Di?

- (l) Karl has 4 cheese bugs each with 4 legs missing. Harry has 5 cheese bugs each with 3 legs missing and one with the right number of legs. Between them their cheese bugs have 189 legs. How many legs should a cheese bug have?
- (m) Mrs Mildew had 6 note pads with 10 pages removed from each. Mr Mildew had twice as many note pad pages as his wife plus 12 more. If Little Miss Mildew had 30 pages more, she would have as many as both her parents put together. The dog has 2 pads with all their pages. The family (including the dog) has 166 note pad pages. How many pages in a pad?
- (n) Misery Guts thought of a number, divided it by 3, added 7, multiplied by 5, then subtracted 18. But he wouldn't tell anyone what he got.
- (o) Pasha thought of a number, added 2, multiplied by 3, subtracted the number he started with, multiplied by 2, added 6, then added a number which was 4 more than twice the number he started with. He got 58. What number did he start with?

Solve

- Q51 Bertrand thought of a number, added 1, then multiplied the result by 2, then added 3, then multiplied by 4, then added 5, then multiplied by 6. This gave him 534. What number did he start with?
- Q52 Magpie thought of a number, added 7, then multiplied the result by 2, then subtracted 3, then subtracted twice the number she started with. What number did she end up with?
- Q53 Sith asked Jabba to think of a number, then to multiply it by -4 , then to add 12, then to divide by 2, then to add twice the number he started with, then to subtract 2. He then told Jabba that he had ended up with 4. Use algebra to explain how he knew.

Revise

Revision Set 1

- Q61 Cowboy thought of a number, added 5, then multiplied the result by 4. He then added the number he started with. This gave him 80. What number did he start with?
- Q62 Solve $2(m + 6) + 3(m + 3) = 41$
- Q63 Slimy had some slugs. On Friday she had 6 more than on Thursday. On Saturday she had 4 times as many as on Friday. On Sunday she had 12 less

than on Saturday. On Monday all the ones she had on Thursday had escaped leaving her with 51. How many did she have on Thursday?

Answers

- Q1 (a) 1 (b) 7 (c) 4 (d) 1
- Q2 (a) 4 (b) 5 (c) 6 (d) 11 (e) 4 (f) 3 (g) 6 (h) 0
- Q3 (a) $3a + 6$ (b) $5x - 15$ (c) $-8c + 8$ (d) $-3s - 12$ (e) $-4d + 8$ (f) $4a + 18$
(g) $4a - 6$ (h) $x - 14$ (i) $-2n + 6$ (j) -3 (k) 3 (l) $2w - 5$
(m) $3a - 3$ (n) $-2a + 11$ (o) $r - 1$ (p) 2 (q) $4a - 6$ (r) $8a - 9$
(s) $-7p - 2$ (t) $5d - 16$ (u) $-2a + 15$ (v) $3d - 11$ (w) $3x - 16$ (x) $23a - 41$
- Q4 (a) 5 (b) 9 (c) 12 (d) -2 (e) 17 (f) 10 (g) 1.5 (h) -2.4 (i) -3 (j) 5.5
(k) 3 (l) 11 (m) 9 (n) 5 (o) 9 (p) 1
- Q5 (a) 9 (b) 9 (c) 13 (d) ? (e) 23 (f) 10 (g) - (h) 15 (i) 140 g (j) - (k) 50
(l) 22 (m) 14 (n) - (o) 6
- Q51 8 Q52 11
- Q61 12 Q62 4 Q63 13