M1 Maths

A2-1 Writing Equations

- solve problems by modelling them with equations

Summary   Learn   Solve   Revise   Answers

Summary

Some problems involve a relation in which we know the value of the dependent variable and have to find a value for the independent variable.

We could write the relation and substitute to get an equation, but in practice it is often easier to write the equation straight off.

What we do is decide what quantity needs to be found, i.e. the unknown; then we decide on an abbreviation for it and state what the abbreviation means; then we decide what operations were performed on the unknown and write them to make the equation; then we solve the equation as normal.

Learn

Getting Equations from Relations – a Recap

Consider this problem:

Josh thought of a number, multiplied it by 4, then subtracted 5. He ended up with 19. What number did he start with?

The relation between the number he ended up with and the number he started with is:

number he ended up with = number he started with × 4 – 5

This can be written

$$z = a \times 4 - 5$$

where z is the number he ended up with and a is the number he started with.

We are told that $z = 17$, so we can substitute 17 for z to get

$$17 = a \times 4 - 5$$

This can then be solved to get $a = 6$. Thus the number he started with was 6.

Writing Equations straight from the Problem

In practice, with this type of problem, we generally wouldn't worry about the relation – we would go straight from the problem to the equation. We decide what it is that we need to find out. In this case it is the number he started with. We give it an abbreviation, say a.

Reading the problem, we see that a was multiplied by 4, then 5 was subtracted and the result was 19. In abbreviated form

$$a \times 4 - 5 = 17.$$

This is the same equation we had before, but obtained directly, rather than via a relation. As before, solving it gives $a = 6$, so we know that Josh started with 6.

Standard Lay Out

To lay this out properly, we would go through a number of steps.

Firstly, decide what it is you need to find out (the unknown) and decide on an abbreviation for it. These two steps will form the first line of your working. In the case above, the first line will read:

> Let the number he started with be a

Then write down what was done to the unknown and what the result was in the form of an equation. In the case above, the second line will read

$$a \times 4 - 5 = 19$$

Then solve the equation. The whole solution will look like this:

Let the number he started with be a

$$
\begin{array}{rl}
a \times 4 - 5 &= 19 \\
{\scriptstyle +5 \quad +5} & \\
a \times 4 &= 24 \\
{\scriptstyle \div 4 \quad \div 4} & \\
a &= 6
\end{array}
$$

← Standard layout

Important Advice on Practice

The art of writing and solving equations is one of the most important things you will ever learn in maths. You will use it in just about every algebra module from here on – and some of the non-algebra modules too. It takes quite a bit of practice to get automatic at it, so it is important to get plenty of practice – even after you feel you've got the hang of it. The equations in this module will be relatively simple, but they get more involved in later modules. If you can become totally automatic with the easier ones here, you will have no trouble with the more complicated ones later on. This module contains 33 practice questions. Please do them all.

Important Advice on Laying Out

Also, you will be required to use the standard layout above in tests from here on. Because we are starting with fairly simple equations, you will be able to do many of the questions in this module in your head without any laying out. However, please still use the full laying out on every question. That way it will become automatic and you will be able to give all your attention to the equation solving when it gets more complicated.

Practice

Q1   Solve the following just by writing and solving an equation. Don't worry about the relation. But do show full working, starting each question with *Let …*

(a)   Ariana thought of a number, multiplied it by 3, then subtracted 7. She ended up with 26. What number did she start with?

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(b)   Bronson thought of a number and multiplied it by 2 and added 5. This gave him 19. What number did he start with?

(c)   Cyril thought of a number, then divided it by 3. This gave him 9. What number did he start with?

(d)   Adnap thought of a number, multiplied it by 2, then added 5. He ended up with 17. What number did he start with?

(e)   Beazle thought of a number, then added 5, then multiplied by 3. If he ended up with 39. What number did he start with?

Be careful with (e). If we wrote $b + 5 \times 3 = 39$, the equation would be wrong, because the order of operations rules tell us that we have to multiply before we add, so the equation would be $b + 15 = 39$, giving us $b = 24$ – not correct. As Beazle added first, then multiplied, we have to put the $b + 5$ in brackets like this: $(b + 5) \times 3 = 39$ to show that the addition was done first.

If we wrote $b + 5 \times 3 = 39$, then undid it ignoring the order of operations rules, we would actually get the right answer, but only because we would have made two mistakes which cancel each other out. Our working would be incorrect. Also, in more complicated equations that you will meet later, the mistakes may not cancel each other out.

(f)   Cecile thought of a number, divided it by 3, then subtracted 4. She ended up with 7. What number did she start with?

(g)   Dory thought of a number, but then forgot what it was.

(h)   Melissa wrote a number on a piece of paper. She then wrote a second number 4 times the first number, then a third number 7 less than the

second. If the third number was 13, what was the first number?

(i) Nagimok had some fish. His friend gave him 12 more. Then he had 30. How many did he start with?

(j) Ozymandias put some rabbits in a cage. Later, there were 4 times as many in there, 20 in fact. How many did he put in?

(k) On Monday, Quentin had some warts. By Thursday he had 3 times as many. By Saturday he had 8 more than on Thursday giving him a grand total of 35. How many did he have on the Monday?

(l) Ratfink planted a tree. A week later it had grown 12cm. During the next week it doubled its size. The week after that it shrunk 4 cm making it 40 cm tall. How tall was it when he planted it?

(m) Tabitha is 7 years older than Taffy, who is half as old as Tinkerbell. If Tabitha is 40, how old is Tinkerbell?

Note: with questions like these, going straight to the equation can be difficult. It can be easier to write something like this:
Let Tinkerbell's age be a. Then Taffy's age is $a \div 2$ and Tabitha's age is $a \div 2 + 7$. So $a \div 2 + 7 = 40$.

(n) Uriah keeps frogs. His brother has 6 less than him. His sister has a quarter as many as his brother. If his sister has 10, how many does Uriah have?

(o) Vladimir builds battle ships. In 1991 he built 1 more than in 1990. In 1992 he built three times as many as in 1991. If he built 3 in 1992, how many did he build in 1990?

(p) Xavier thought of a number, added 3, multiplied the result by 4, then subtracted 5. This gave him 27. What number did he start with?

(q) Yippee thought of a number, divided it by 4, added 7, then multiplied by 2 to get 20. What number did she start with?

(r) Zebedee tried to think of a number but couldn't.

Q2 Solve the following by writing and solving an equation. Show full working

(a) Murgatroid thought of a number, divided it by 5, subtracted 6 then multiplied the result by 2. If she ended up with –8, what number did she start with?

(b) Margot thought of a number, subtracted 16, then multiplied by 4, then added 7, giving herself 25. What number did she start with?

(c) Morgan thought of a number, halved it, added 12, subtracted 20, then multiplied it by 4. If he ended up with 26, what number did he start with?
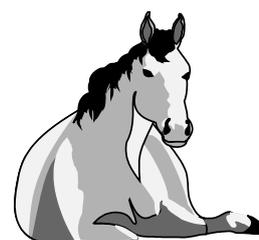
(d) Minnie planted some weeds in her garden. A week later they were still there and 12 more had grown. In disgust she then pulled out half of all the weeds, leaving 27. How many did she plant?

(e) Moriarti had 4 identical packs of Christmas cards and 10 loose ones. If this totalled 42 cards, how many were in each pack?

(f) Mibbs had 8 identical boxes of guinea pigs with 3 missing out of each. If she had 32 guinea pigs altogether, how many should there be in a box?

(g) Mushroom placed a bet on the horses. He then increased the bet by $12. Later, he decided to double his bet to $40. How much did he bet in the first place?

(h) On Monday Miss Marples made some magic marbles. By Tuesday she had 5 times as many. On Wednesday she lost 27 of her marbles. On Thursday she lost half of what she had left. On Friday she made 22 more then lost 15. If she ended up with 21, how many did she make on Monday?

(i) On Tuesday Mia had twice as many sore throats as on Monday. How many did she have on Monday?

(j) Maud, who is 84, is twice as old as Martina who is 14 years older than twice Mandy's age. How old is Mandy?

(k) Maxwell had a pile of peas which he didn't really like. He divided them into 5 equal piles and gave 4 of the piles away. Then he put 150 peas down the toilet and hid 80 in the DVD player. The dog then ate half of what he had left, leaving him with 125. How many did he start with?

(l) Griffin thought of a number. He doubled it, then subtracted 4. This gave him 3. What number did he start with?

(m) Arthur thought of a number, added 6, then multiplied by 4, then subtracted 12. This gave him 30. What number did he start with?

(n) Tammy had some money. She doubled it, then earned another $8.50, then doubled it again. If she ended up with $27, how much did she start with?

(o) Nadia's 2.7kg rabbit weighs 3 times as much as her tortoise, which weighs 0.4kg more than her 2 budgies put together. How much does one budgie weigh?
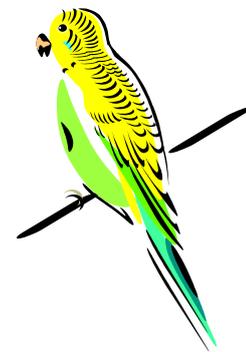
How much did you enjoy that exercise? Pick one of these faces.

Q51 Jack's doctor told him he should walk more. On Monday, he walked the amount his doctor recommended. On Tuesday, he walked 5 km more than his doctor recommended. On Wednesday, he walked 3 km less than recommended. Over the three days he walked 38 km. Write and solve an equation to work out how many kilometres his doctor recommended him to walk per day?

Q52 Katie gave ¼ of her money to Shana, then 20% of what was left to Dora. This left her with $168.

(a) How much did she have before she gave any away?

(b) How much did she give Dora?

Q53 Harry had half as much money as Deb and Deb had half as much money as Harry. How much did they have between them?

Revise

Revision Set 1

Q61 Solve the following by writing and solving an equation. Show working.

(a) Gonzo thought of a number, added 6, multiplied the result by 3, then subtracted 13. This gave him 71. What number did he first think of?

(b) If Harry had twice as much money and another $20, he would have $87. How much does he have?

(c) Fiona has 65 beans. If she stole half of Katie's beans and another 20 beans from Doug, she would then have 200. How many did Katie have before Fiona stole some?

(d) Prudence is 12 years older than Randy, who is half as old as Dick. If Prudence is 36, how old is Dick?

Answers

Q1	(a) 11	(b) 7	(c) 27	(d) 6	(e) 8	(f) 33
	(g) -	(h) 5	(i) 18	(j) 5	(k) 9	(l) 10 cm
	(m) 66	(n) 46	(o) 0	(p) 5	(q) 12	(r) -
Q2	(a) 10	(b) 20.5	(c) 29	(d) 42	(e) 8	(f) 7
	(g) $8	(h) 11	(i) 0	(j) 14	(k) 2400	(l) 3.5
	(m) 4.5	(n) $2.50	(o) 0.25 kg			

Q51 12 km
Q52 (a) $280 (b) $42
Q53 $0

Q61	(a) 22	(b) $33.50	(c) 230	(d) 48