

#### Summary

If we substitute a value for the independent variable in an equation, we get an implicit equation with an unknown which we can solve.

To solve it, we look at what operations were performed on the unknown and use inverse operations to undo them in the reverse order, starting with the last to be performed.

When solving equations, we often have to write the name of the unknown several times. To save time and space, we can use a single-letter abbreviation for the unknown. We can also omit multiplication signs as long as this won't cause confusion and as long as it is not before a numeral. We can also express division using fractions.

Learn

## Substituting for the dependent variable to find the value of the independent variable

Consider a simple formula for the relation between Dave's age and his younger sister Meg's age. Dave is 4 years older than Meg, so we have:

Dave's age = Meg's age + 4

In this formula, Dave's age is the dependent variable because it is by itself on the left of the = sign; Meg's age is the independent variable because it is on the right side of the = sign.

In Module A1-5 we learnt how to substitute for the independent variable, Meg's age, so that we could find the value of the dependent variable, Dave's age. We will now learn to substitute for the dependent variable, Dave's age and find the value of the independent variable, Meg's age.

Let's suppose Dave's age is 20. The substitution part is just the same as before, except that we take out the dependent variable and substitute it with its value.

$$Dave's age = Meg's age + 4$$
$$20 = Meg's age + 4$$

Note that we write both sides of the formula on the second line. We cannot omit one side like we did when substituting for the independent variable.

This second line is called an **equation** and the process of finding the value of the independent variable is called **solving the equation**. The variable, Meg's age, is, strictly speaking, no longer a variable because it can only be one value, 16. So we call it an **unknown**. Actually, we know it is 16, but that is only because this is a very simple equation and it's very easy to see that the solution will be 16. But in most situations, we won't know the value of the unknown until we go through the process of solving the equation. So calling it the unknown is quite appropriate.

The process we go through is shown in this diagram.



[Strictly speaking, substituting for the independent variable like we did in Module A1-5 also produces an equation, but we generally don't call that an equation and the process of finding the value of the dependent variable isn't called solving.]

#### Solving the equation

Ok, our equation is:

20 = Meg's age + 4

This says that Meg's age + 4 is equal to 20. We want to know what *Meg's age* is before the 4 is added on. What we do is just take the 4 back off. *Meg's age* without the 4 will be 4 less than 20:

20 = Meg's age + 4-4 = -4 16 = Meg's age

So Meg's age is 16.

The best way to think of this is that we subtract 4 from the right-hand side of the equation in order to get *Meg's age* by itself. Then, of course, we have to subtract 4 from the left-hand side so that the left-hand side will still be equal to the right-hand side. In other words, we subtract 4 from both sides.

It doesn't matter which side the unknown is on: we just subtract the same number from both sides.

#### Practice

- Q1 Solve these equations, using the lay-out above. You can use your calculator where necessary.
  - (a) 30 = 4 + Meg's age
  - (b) 22 = 7 + Meg's age
  - (c) *Meg's age* + 2 = 14
  - (d) *Meg's age* + 8 = 40
  - (e) *Fifi's age* + 34 = 72
  - (f) 36 = Fifis age + 1
  - (g) fare + 5 = 35
  - (h) 31.9 = fare + 17.4
  - (i) length + 139.68 = 323.1
  - (j) number of tortoises +58 = 427

Subtracting 4 undoes the effect of adding 4 and takes us back to the original number. We can call this **solving by undoing**.

In the same way, adding four would undo the effect of subtracting 4. So if the equation were

*Meg's age* -4 = 15

We would solve it by adding the 4 back on, like this:

15 = Meg's age - 4+4 = +4 19 = Meg's age Also, division undoes multiplication and multiplication undoes division. Operations that undo each other are called **inverse operations**. So addition and subtraction are inverse operations; and multiplication and division are inverse operations. If we had Meg's age  $\times 4 = 24$ , we would have to divide by 4 to get her original number:

```
Meg's age \times 4 = 24
\overset{\div 4}{} \overset{\div 4}{}Meg's age = 6
```

And if we had Meg's age  $\div 4 = 8$ , we would have to multiply by 4 to get her original number:

 $Meg's age \div 4 = 8$   $\times 4 \qquad \times 4$   $Meg's age \qquad = 32$ 

#### Practice

- Q2 Solve the following equations by undoing using inverse operations. Use the same lay-out.
  - (a) Meg's age + 6 = 17
  - (b) Meg's age 2 = 5
  - (c) *Fifi's age* -37 = 88
  - (d) Fifi's age  $\times 5 = 65$
  - (e) Fifi's  $age \div 3 = 29$
  - (f) number of cars  $\times$  67 = 14 271
  - (g)  $capacity \div 4.3 = 42.87$
  - (h) Harry's mass 9.3 = 22.8
  - (i)  $volume \times 1.44 = 27.36$
  - (j) *starting number* + 4.3 = 42.87

All the writing will seem quite tedious, but stick with it for now. Later on in this module, we will switch to something much shorter.

#### **Two-step Equations**

Now let's make it a bit more challenging.

Take the relation  $fare = distance \times 3 + 5$ 

If fare is 26, then, when we substitute, we get  $26 = distance \times 3 + 5$ 

This is a two-step equation in that two things have been done to the unknown: first it was multiplied by 3, then 5 was added.

To solve it, we undo the two steps in turn. But we undo the last one that was done first.

First we take off the 5 to find out what the distance would have been after it was multiplied by 3, but before 5 was added; then we divide by 3 to find out what it was to start with. Remember we do these two steps to both sides to keep them equal. The working will look like this:

 $26 = distance \times 3 + 5$   $^{-5} \qquad ^{-5}$   $21 = distance \times 3$   $^{\div 3} \qquad ^{\div 3}$  7 = distance

#### Note on laying out - Equals signs underneath one another

Note that in the examples above, the = signs are all underneath each other in a vertical line. This is a mathematical convention and you should do the same when solving equations. The reason is that it keeps the two sides of the equations separated and makes the process easier to follow.

#### Practice

Q3 Solve these equations.

- (a)  $26 = distance \times 3 + 5$
- (b)  $17 = distance \times 4 + 1$
- (c)  $19 = distance \times 3 5$
- (d)  $distance \times 3 + 5 = 29$
- (e)  $distance \times 2 + 11 = 33$
- (f)  $age \times 3 4 = 14$
- (g)  $length \div 3 + 6 = 10$
- (h) number of ostriches  $\div 5 11 = 23$
- (i)  $mass \times 4.6 + 19.1 = 78.9$
- (j) Joe's starting number  $\div 26 + 115 = 120$
- (k) original price  $\times 23.6 59.4 = 212$
- (l) time taken  $\times 3 \div 5 = 4.8$

Now we will combine substituting with solving. Here is an example:

```
If fare = distance \times 4 + 3, find the distance if the fare is 23.
```

 $fare = distance \times 4 + 3$   $23 = distance \times 4 + 3$   $-3 \qquad -3$   $20 = distance \times 4$   $\div 4 \qquad \div 4$  5 = distanceSo the distance is 5

### Practice

Q4 Solve the following by substituting for the dependent variable and solving the resulting equation.

- (a) If  $fare = distance \times 4 + 3$ , find the distance if the fare is 27.
- (b) If  $pay = hours worked \times 20 12$ , find the hours worked if the pay is 68.
- (c) *time taken* = *number of people*  $\div$  5 + 12. Find the number of people if the time taken is 28
- (d) If  $fare = distance \times 2 + 7$ , find the distance you can go if you pay \$27.
- (e) If  $fare = distance \times 3.6 + 2.4$ , how far can you travel for \$56.40?
- (f) A labourer is paid according to the formula  $pay = hours worked \times 12$ . How many hours would he have to work for \$90?
- (g) An electrician charges according to the formula  $charge = time \times 20 + 25$ where *charge* is in dollars and *time* is the time spent in hours. What length visit would attract a charge of \$115?
- (h) A retailer makes weekly profit on turnover according to the formula *profit = turnover* ÷ 5 \$400
   What turnover does she need to make to get a profit of \$1000?

#### Algebraic Shorthand

Writing out the solution of equations like this involves writing the name of the unknown several times. In more involved equations you might end up writing it even more times.

For this reason, people often abbreviate names of variables and unknowns to single letters. You may already have been doing this, but, whether you have or not, feel free to do it from now on.

We often use the initial letter of the name of the variable or unknown. For instance, if the variables are *cost* and *distance*, we might use the letters c and d. This is not

necessary. We could use p and h or z and t, but using initial letters helps us to remember what the letters stands for.

Note, however, that when we use letters, we use a single letter, not two or three letters. So *distance in kilometres* can be represented by d, but should not be represented by *dist or dik*. You will see the reason for this shortly.

Now if you are solving an equation for yourself and no one else will be looking at your solution, you don't need to worry about explaining what your letters mean. But if you are writing up your solution for others to look it, like in a test, you should include an explanatory note before you first use the single letter. The standard way of writing this note is as follows:

Let the distance be d.

Now, if we have a question like this:

If  $cost = distance \times 4.5 + 15$ , find the distance if the cost is 80

the solution would look like this:

Let the cost be c and let the distance be d

```
c = d \times 4.5 + 15

80 = d × 4.5 + 15

-15 -15

65 = d × 4.5

+4.5 +4.5

14.4 = d
```

So the distance is 14.4.

Another form of shorthand used in algebra is the omission of multiplication signs.

 $2 \times b$  can be written as 2b  $s \times t$  can be written st(This explains why you should always use a single letter for variable names: if you used dik, it could be read as  $d \times i \times k$ .)  $5 \times (t+2)$  can be written 5(t+2) $4 \times \pi$  can be written  $4\pi$ .

The only time that you can't do this is when the multiplication sign is followed by a numeral: we cannot abbreviate  $2 \times 4$  to 24 (for obvious reasons) and we don't abbreviate  $h \times 2$  to h2 (though it can of course be written as 2h).

No other sign (+, – or  $\div$ ) can be omitted, so we know that, if a sign is omitted, it must be a ×.

Also, as you know from your studies of fractions, division can be written as 'over'. For example  $(x + 2) \div 7$  can be written  $\frac{x+2}{7}$ .

#### Practice

リリリリリリリリ

- Q5 Solve the following using single-letter abbreviations for variables/unknowns. Start with a note to say what abbreviations you are using.
  - (a) A plumber charges according to the formula *charge in dollars* = *time spent in hours*  $\times$  36 + 28. How long did he spend on a job for which he charged \$127?
  - (b) The charge for the hire of a generator is worked out by the formula charge = the number of days the generator is hired for × \$17.20 + \$45 Sam paid \$268.60. How long did he hire the generator for?
  - (c) The number of people needed to lift a car is given by the formula number of people = mass of car in kilograms ÷ 80 + 2. What is the mass of a car that 9 people can just lift?
  - (d) The number of matches needed to make a row of house shapes is given by the formula *number of matches* = *number of houses*  $\times$  4 + 1. How many houses can be made with 89 matches?



- (e) Fiona thought of a number. She multiplied it by 3 and added 7. So *the number she ended up with = the number she started with × 3 + 7*. If the number she ended up with was 58, what was the number she started with?
- (f) Sarah thought of a number, multiplied it by 7 then subtracted 13. If she ended up with 148, what number did she start with? [Do this the same way as the last one.]
- (g) Pasha thought of a number, divided it by 5, then added 17. He ended up with 29. What number did he start with?
- (h) The formula for the number of matches needed to make a row of adjacent octagons is  $m = n \times 7 + 1$ , where *m* is the number of matches and *n* is the number of octagons. How many octagons are there in a row that takes 155 matches to construct?

Note that when the abbreviations are defined in the question, as in (h) above, you don't have to define them again in your working -just use them.

(i) The delivered cost of ti-tree mulch with added chook poo from Suzie's Garden Centre is calculated by the formula c = 88m + 48, where *c* is the cost in dollars and *m* is the quantity in cubic metres. Ahmed's delivery cost him \$444. How much mulch did he buy?



Q6 Solve the following equations:

(a) $2w + 5 = 21$	(b) $k \div 2 + 5 = 11$	(c) $5a - 3 = 32$
(d) $\frac{x}{4} + 1 = 8$	(e) $d \times 5 - 23 = 7$	(f) $\frac{t}{3} - 1 = 7$
(g) $5z + 23 = 98$	(h) $h \div 5 - 3 = 11$	(i) $p + 7 - 12 = 13$

#### **Multi-step Equations**

Some equations can have more than two operations. So they take more than two steps to solve them. Let's look at  $k \times 5 \div 2 + 7 = 22$ .

Here k was multiplied by 5, then the result divided by 2, then 7 was added to end up with 22. To solve it we have to work our way back undoing the operations in the reverse order from the order they were done in. The solution would look like this:

$k \times 5 \div 2$ +	+7=22
	-7 -7
$k \times 5 \div 2$	= 15
$\times 2$	$\times 2$
$k \times 5$	= 30
÷ 5	$\div 5$
k	= 6

#### Practice

$\mathbf{Q7}$	Solve the	se multi-step equations:		
	(a) $w \times$	$4 \div 3 + 5 = 9$	(b)	$5x \div 3 - 1 = 19$
	(c) $\frac{r}{2} \times$	7 + 9 - 15 = 22	(d)	$\frac{4a}{5} + 1 = 9$
	(e) ( <i>x</i> –	$(4) \times 3 + 5 = 23$	(f)	$(a + 3) \div 2 - 11 = 0$
	(g) ( <i>p</i> –	$(4) \times 3 = 27$	(h)	$(w \times 3 + 6) \times 4 = 60$
	(i) $\frac{w+7}{5}$	-1 = 2	(j)	$\frac{2a+5}{3} + 9 = 14$
	(k) ( $d$ +	$(2) \times 4 - 10 = 6$	(l)	$(5h - 7) \div 2 + 6 = 15$

#### Order of Operations

Up to now, the rules about order of operations haven't really been an issue. But in some equations we have to take them into account. Remember that multiplication and division are worked out before the addition and subtraction are performed; anything in a bracket is worked out first.

If we have the equation  $4 + s \times 3 = 28$ , then we have to know what was done to the unknown first and second, so we can undo them in the reverse order. In this case *s* was multiplied by 3 first and the result was added to 4 afterwards to make 28. So we have to take the 4 off first, like this:

$$4 + s \times 3 = 28$$
  
$$-4 \qquad -4$$
  
$$s \times 3 = 24$$
  
$$\div 3 \qquad \div 3$$
  
$$s = 8$$

If we have 10(p-3) = 120, then we see that the p-3 in the bracket is done first, then the result is multiplied by 10. So we undo the  $\times$  10 first, like this:

In some equations there can be more than two operations performed on the unknown, like in  $23 + 8 \times (2b - 4) = 33$ . We still work out the order in which the operations were performed and undo them in the reverse order, like this:

$$23 + 8 \times (2b - 4) = 63$$
  
-23 -23  
$$8 \times (2b - 4) = 40$$
  
 $\div 8$   
$$2b - 4 = 5$$
  
+4 +4  
$$2b = 9$$
  
 $\div 2$   
$$b = 4.5$$

# In fact, when solving any equation, you should look at the order in which the operations were performed on the unknown and then undo them in the opposite order.

With the first equations we solved, the order the operations were done in was the order they were written in. But this won't always be the case, so from now on, always use this procedure – look at the order the operations are done in, then undo them in the reverse order.

#### Practice

Q8 Here are some equations where you can start to get used to using the procedure in red above. Reading the formula, abbreviating the variables and substituting are all done for you. All you have to do is solve the equation.

(a)  $3 + s \times 4 = 15$ 

(b) 5x - 1 = 19

(c)  $1 + \frac{r}{2} = 5$ (d) 2(h+1) = 12(e) 3(p-4) = 27(f)  $(a+3) \div 2 = 5$ (g)  $\frac{a+5}{2} = 12$ (h)  $14 + \frac{3w+7}{5} = 25$ (i)  $(3w+6) \times 4 = 60$ (j)  $1+k \div 2-3 = 9$ (k)  $(s+2) \times (2+2) - 11 = 9$ (l)  $(3+5x) \div 2+7 \times 3 = 45$ (m)  $8 \times (b-4) - 23 = 33$ (n)  $[5(\frac{f}{3} + 19) - 1] \div 2 - 4 = 53$ 

With the following questions, you should abbreviate the variables, substitute the value(s) and do the calculations/solve the equation. Don't forget to use the procedure in red above.

#### Practice

**Q**9 Dwayne Pipe the plumber charges for labour according to the formula:  $charge = hours spent \times \$24$ How much will he charge for jobs that take: (a) 2 hours (b)  $4^{1/4}$  hours How many hours would he have worked if he charged: (c) \$72 (d) \$54 The cost for a limousine ride is given by the formula Q10 cost in dollars = distance in kilometres  $\times 4.5 + 15$ How far can you go for \$80? Q11 An electrician charges according to the formula:  $charge = 75 + time \, spent \times 60$ where *charge* is in dollars and *time spent* is in hours What length visit would attract a charge of: (a) \$435 (b) \$180 Q12 A taxi driver charges:  $4 + kilometres travelled \times 1.20$ How far will she take you for: (a) \$11.20 (b) \$18.40 Q13 The fuel used on a journey is given by:  $fuel = 2 + \frac{distance\ travelled - 5}{2}$ 10 where *fuel* is in litres and *distance travelled* is in kilometres How far could you go on the following amounts of fuel? (a) 30 L (b) 67.2 L

Q14 The number of steel rods needed to build a structure is given by  $number \ of \ rods = 6 \times length - 20$  where length is in metres How many would be needed to build structures of lengths: (a) 10m (b) 12.5m What length structures would need these numbers of rods: (c) 100 (d) 214

#### More Than One Independent Variable

As you know, some formulae have more than one independent variable. With these, if we substitute for all the independent variables, then we can find the value of the dependent variable.

```
area = length \times width length = 10, width = 4
area = 10 \times 4
area = 40
```

We can also substitute for the dependent variable and solve the equation to find the value of an independent variable. BUT we must substitute for all the other independent variables as well so that there is only one unknown.

For example, if

```
volume = length \times width \times height and volume = 36, length = 6 and width = 2
```

then 36 = 6 Х 2× height ÷6 ÷6 6  $2 \times height$ = ÷2 ÷2 3 height =

Answer the following questions which come from formulae with more than one independent variable.

#### Practice

Q15	The area of a rectangle is given by the formula: $area = length \times width$ ,				
	Find the lengths of rectangles with the following areas and widths:				
	(a) $area = 40 \text{ m}^2$ ; $width = 8 \text{ m}$				
	(b) $area = 12 \text{ m}^2$ ; $width = 5 \text{ m}$				
Q16	The perimeter of a rectangle is given by perimeter = $(length + width) \times 2$				

What is the length of a rectangle with perimeter 26 m and width 3 m?



Find the height of a box with length = 22 cm; width = 13 cm; volume =  $411 \text{ cm}^3$ 

Solve

- Q51 The perimeter, p, of a regular hexagon with side length s is given by the formula p = s + s + s + s + s + s. Find the side length of a regular hexagon with perimeter 15 cm.
- Q52 The area of a circle is given by the formula  $A = \frac{\pi}{4}d^2$ , where A is the area, d is the diameter and  $\pi$  is approximately 3.14. Find the diameter of a circle with area 20 cm<sup>2</sup>.
- Q53 A taxi charges a flag fall of \$3.50 and then \$2.20 per kilometre. Write the relation between distance and cost as a formula, then use the formula to find how far Gerry went if his fare was \$41.78.
- Q54 The equation  $x^2 + x = 6$  has two solutions. In other words there are two values for x which make the equation true. Find one or both of them. Guess and check is probably the best method. [Hint: one of the solutions is a negative number.]
- Q55 a + b = 12; a b = 2. Using guess and check or otherwise, find the values of a and b.
- Q56 Peter and Mary have 87 books between them. Peter has 11 more than Mary. How many does Mary have?

#### Revise

#### **Revision Set 1**

- Q61 Solve the following, showing working.
  - (a) Meg's age 12 = 31
  - (b)  $Danny's age \times 6 = 84$
  - (c)  $s \times 4 18 = 16$

- (d)  $(time + 10) \div 2 + 28 = 143$
- (e) 4(2a+5) = 108
- (f)  $\frac{3h+5}{6} = 43$
- (g) If *fee* = *hours worked*  $\times$  25 + 45, find the hours worked if *fee* = 195
- (h) The volume of a box is given by  $V = l \times w \times h$ , where V is the volume, *l* is the length, *w* is the width and *h* is the height. If *l* is 12 cm, *h* is 5 cm and V is 240 cm<sup>3</sup>, find *w*.
- (i) The simple interest, *i*, on a deposit of \$*p* left in the bank for *t* years at an interest rate of r% is given by  $i = p \times r \times t \div 100$ . How long would you have to leave \$200 in the bank at an interest rate of 5% to get \$80 in interest?

Answers						
Q1	(a) 26	(b) 15	(c) 12	(d) 32	(e) 38	(f) 35
	(g) 30	(h) 14.5	(i) 183.42	(j) 369		
Q2	(a) 11	(b) 7	(c) 125	(d) 13	(e) 87	(f) 213
	(g) 184.341	(h) 32.1	(i) 19	(j) 38.57		
Q3	(a) 7	(b) 4	(c) 8	(d) 8	(e) 11	(f) 6
	(g) 12	(h) 170	(i) 13	(j) 130	(k) 11.5	(l) 8
$\mathbf{Q4}$	(a) 6	(b) 4	(c) 80	(d) 10	(e) 15	(f) 7.5
	(g) 4.5 h	(h) \$7000				
Q5	(a) 2.75 h	(b) 13 days	(c) 560 kg	(d) 22	(e) 17	(f) 23
	(g) 90	(h) 22	(i) $4.5 \text{ m}^3$			
Q6	(a) 8	(b) 12	(c) 7	(d) 28	(e) 6	(f) 24
	(g) 15	(h) 70	(i) 18			
$\mathbf{Q7}$	(a) 3	(b) 12	(c) 8	(d) 10	(e) 10	(f) 19
	(g) 13	(h) 3	(i) 8	(j) 5	(k) 2	(l) 5
$\mathbf{Q8}$	(a) 3	(b) 4	(c) 8	(d) 5	(e) 13	(f) 7
	(g) 19	(h) 16	(i) 3	(j) 22	(k) 3	(l) 9
	(m) 11	(n) 12				
Q9	(a) \$48	(b) \$102	(c) 3	(d) 2.25		
Q10	14.4 km					
Q11	(a) 6 h	(b) 1.75 h				
Q12	(a) 6 km	(b) 12 km				
Q13	(a) 285 km	(b) 657 km				
Q14	(a) 40	(b) 55	(c) 20 m	(d) 39 m		
Q15	(a) 5 m	(b) 2.4 m				
Q16	10 m					
Q17	1.44 cm					
<b>Q</b> 51	$2.5~\mathrm{cm}$	Q52. 5.05 cm	Q53.	17.4 km	Q54. $x = 2$ a	nd $x = -3$
Q55	a = 7, b = 5	Q56. 38				
Q61	(a) 43	(b) 14	(c) 8.5	(d) 23	(e) 11	(f) 84.3
	(g) 6	(h) 4 cm	(i) 8 years			