M1 Maths

A1-5 Substitution

- substitute for the independent variable in a formula and do the arithmetic to find the value of the dependent variable
- convert from formulae to other forms of a relation
- substitute into formulae with more than one independent variable

Summary Learn Solve Revise Answers

Summary

If we know the value of the independent variable in a formula, we can substitute the value for the variable, then do some arithmetic to work out the value of the dependent variable.

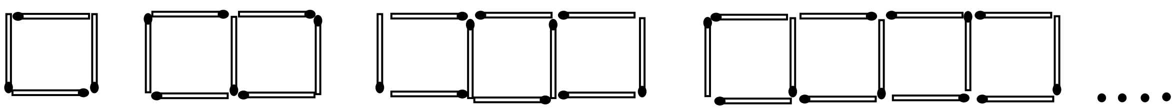
If we substitute for a range of values of the independent variable in a formula, we can produce a set of value pairs and thus convert the formula to a table, graph or set of ordered pairs.

The set of values for the independent variable in a relation is called the domain of the relation.

If a formula has more than one independent variable, we must substitute for all of them before we can find the value of the dependent variable.

Learn

Substituting for the independent variable in a formula



Earlier, we looked at the relation between the number of squares in the designs above and the number of matches needed to make them.

Squares	1	2	3	4	5	6	…
Matches	4	7	10	13	16	19	…

We saw that the relation had a pattern which can be described by the formula:

$$\textit{number of matches} = \textit{number of squares} \times 3 + 1$$

Like any other form of a relation, this formula lets us find one quantity if we know the other.

In a formula, the variable by itself on the left side of the = sign is the dependent variable; the independent variable and some numbers are generally on the right side.

In this module we will learn how to find the value of the dependent variable if we know the value of the independent variable.

To do this we just re-write the right-hand side of the formula, replacing the variable *number of squares* with its value. For instance, if we know that the number of squares is 12, we replace 'number of squares' with 12, like this:

$$\textbf{\textit{number of matches}} = \textbf{\textit{number of squares}} \times 3 + 1$$
$$= 12 \times 3 + 1$$

Then we work out that $12 \times 3 + 1 = 37$, so 37 matches are needed.

The complete working will look like this:

$$\textbf{\textit{number of matches}} = \textbf{\textit{number of squares}} \times 3 + 1$$
$$= 12 \times 3 + 1$$
$$= 37$$

Note that you should always have these three lines, the first line being the formula, the second line being the arithmetic and the third line being the answer.

Replacing the variable with the value is called **substituting**. Just like in a game, where we might take one player out and substitute them with another player, in a relation we take the variable out and substitute it with the value.

Practice

Q1 Use the formula *number of matches* = *number of squares* × 3 + 1 to find the number of matches when the number of squares is:

(a) 7 (b) 60

Q2 Use the formula *number of matches* = *number of squares* × 3 + 1 to find the number of matches required for:

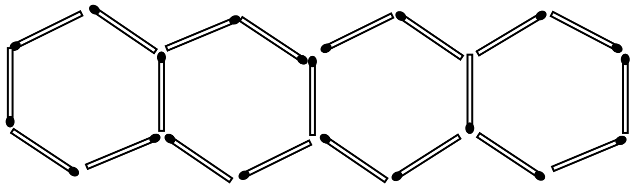
(a) 5 squares (b) 52 squares

Q3 Find the number of matches needed to make the following numbers of squares:

(a) 4 (b) 518

Q4 The number of matches required to make a line of hexagons is given by the formula

number of matches = *number of hexagons* × 5 + 1.



Find the number of matches required for each of the following numbers of hexagons:

(a) 4 (b) 76

Q5 The formula for the perimeter of a rectangle 5 cm wide is *perimeter* = *length* × 2 + 10. Find the perimeters of 5 cm-wide rectangles with each of the following lengths:

(a) 10 cm (b) 1 m

Q6 Use the formula *fare* = *distance* × 2 + 3 to find the fare for each of the following distances:

(a) 5 (b) 19.2

Q7 Amie is a draughtswoman. She charges for jobs according to the time taken. She uses the formula *charge* = 25 + 30 × *time*,
where the charge is in dollars and the time is in hours.

How much would she charge for jobs of the following lengths:

(a) 1 hour (b) 15 minutes

Q8 Use the formula *tension* = *speed* ÷ 2 + 4 to find the tension for each of the following speeds:

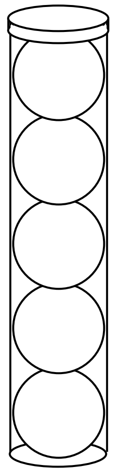
(a) 7 (b) 294

Q9 Use the formula *height* = 1000 – *width* × 3 to find the height for each of the following widths:

(a) 2 (b) 250

Don't forget that you have to multiply *width* by 3 before you take that from 1000 (Remember – multiplication and division are done before addition and subtraction.)

Converting formulae to other forms of a relation



Now that you can find the value of the dependent variable from the formula and the independent variable, you can convert a formula to a set of ordered pairs, a table or a graph.

A company makes plastic tubes to hold up to 5 tennis balls.

The relation between the amount of plastic used and the number of tennis balls the tube is designed for is given by the formula

$$\textit{amount of plastic} = 20 + \textit{number of balls} \times 25.$$

Suppose we want this relation in another form. All we do is find the amount of plastic when the number of balls is 1, 2, 3, 4 and 5, like this:

When *number of balls* = 1, *amount of plastic* = $20 + 1 \times 25 = 45$
When *number of balls* = 2, *amount of plastic* = $20 + 2 \times 25 = 70$
When *number of balls* = 3, *amount of plastic* = $20 + 3 \times 25 = 95$
When *number of balls* = 4, *amount of plastic* = $20 + 4 \times 25 = 120$
When *number of balls* = 5, *amount of plastic* = $20 + 5 \times 25 = 145$

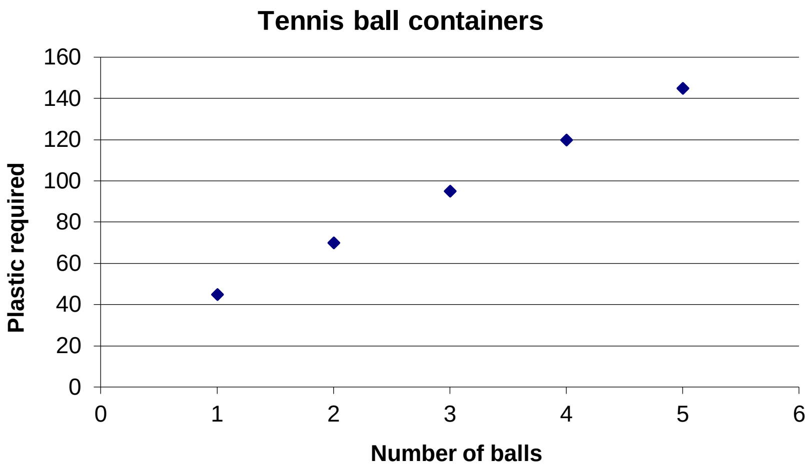
The set of ordered pairs would be:

(1, 45), (2, 70), (3, 95), (4, 120), (5, 145), where the first number is the number of balls and the second is the amount of plastic.

The table would be:

number of balls	1	2	3	4	5
amount of plastic	45	70	95	120	145

The graph might look like this



If you can do the arithmetic in your head, you might skip the first steps and go straight to the set of ordered pairs, table or graph.

Practice

Q10 It has been discovered that the number of passengers carried on a particular route is related to the number of buses that run per day. The number of buses can vary from 6 to 12. The relation can be expressed by the formula

number of passengers = 11 × *number of buses* + 96.

Express this relation as a table, as a set of ordered pairs and as a graph.

Q11 Pam's Paving Company specialises in laying paving around pools. The pavers they use are square with sides of 30 cm. Some customers require only one row of pavers around the pool; some require two rows, so they have a 60 cm-wide paved surround as in the picture; some require 3 rows, and so on. For a 9 m by 4.5 m pool, the total number of pavers required is given by the formula

$\textit{no. of pavers} = 90 \times \textit{no. of rows} + 4 \times (\textit{no. of rows})^2$.

Draw up a table and graph showing the number of pavers required for up to 6 rows.

Domain

In the tennis ball container relation above, you were told what values the independent variable (number of balls) could take: it was 1, 2, 3, 4 or 5. This set of allowed values for the independent variable is called the **domain** of the relation.

In a set of ordered pairs, a table or a graph, the domain is always obvious. When we translate between them, there is one value pair in the new form for each value pair in the old form. But in a formula, the domain is not obvious unless we say what it is. When translating from a formula to another form of a relation, we have to know the domain.

In the practice questions above, the domain for the bus relation was the numbers from 6 to 12. It wasn't specified that it was just the whole numbers, but common sense tells us that you can't drive half a bus, so we assume it is just the whole numbers. Similarly, the domain for the paver question is the numbers from 1 to 6. We assume that only whole pavers are used, so the domain is the whole numbers from 1 to 6.

But in a continuous relation, the domain will include the fractional numbers between the whole numbers as well. Suppose the cost of home-delivered rump steak is given by the formula $\textit{cost} = \textit{mass} \times 20 + 25$, where *cost* is in dollars and *mass* is in kilograms. It is possible to get 1.7 kg delivered, so the relation would be continuous and the domain would be all the numbers from 0 up to the maximum that can be delivered.

To translate this relation to a table, set of ordered pairs or graph, we would need to know what the maximum that can be delivered is. Let's say it's 8 kg. Then the domain is all the numbers from 0 to 8 including the fractional numbers. We say it is all the real numbers from 0 to 8. **Real numbers** is a term we use to mean the whole numbers and all the possible fractions in between.

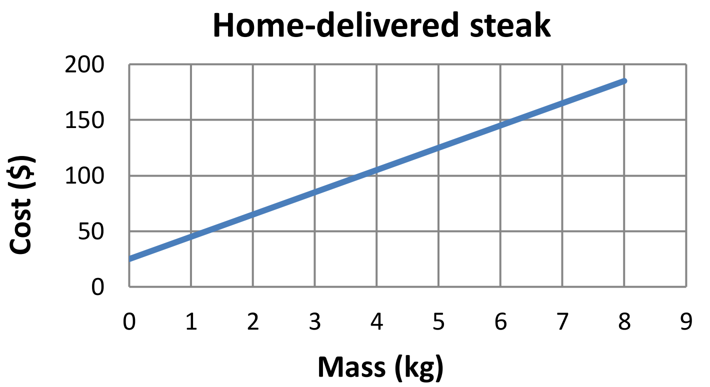
Now of course the relation has an infinite number of value pairs, so we cannot write it fully as a set of ordered pairs or table. Instead we make a sample set of ordered pairs or sample table. We might pick all the whole numbers for example. The relation might then look like:

(0, 25), (1, 45), (2, 65), (3, 85), (4, 105), (5, 125), (6, 145), (7, 165), (8, 185), where the first number is the mass in kilograms and the second number is the cost in dollars.

Or as a table, it might look like this:

mass (kg)	0	2	4	6	8
cost ($)	25	65	105	145	185

As a graph, of course, we can express the relation fully by using a line instead of individual points. It might look like this:



Practice

Q12 The relation between the number of people staying in a room and the cost for the room at the Pelican Lodge Hotel is given by cost = number of people × 40 + 25. This rate applies for up to 6 people.

State the domain for this relation, then present the relation as a set of ordered pairs and as a graph.

Q13 The relation between the number of pizzas delivered and the cost is:
cost = 8 + number × 7. The domain is the whole numbers from 1 to 8 inclusive. Present this relation as a table and as a graph.

Q14 The relation between the fare for a pink taxi and distance travelled is given by the formula *cost* = 5 + *distance* × 3, where *cost* is in dollars and *distance* is in kilometres. The domain is the real numbers from 0 to 20. Present this relation as a sample table and as a graph.

Q15 The relation between the area of a circle and its radius is given by

$area = 3.14 \times radius^2$, where *area* is in cm^2 and *radius* is in cm.

Present this relation as a set of ordered pairs, as a table and as a graph. Use the real numbers from 0 to 100 as the domain.

Substituting into formulae with more than one independent variable

So far we have dealt with relations which have one independent variable and a dependent variable. For instance, the number of matches required to make various numbers of squares is given by the relation

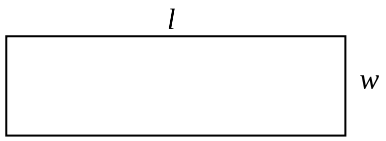
number of matches = *number of squares* × 3 + 1;

the perimeter of a square is given by the relation

perimeter = *side length* × 4.

But we can have relations with two or more independent variables (though there will only ever be one dependent variable). An example is the relation between area, length and width of a rectangle:

area = *length* × *width,*



where the area in square centimetres and the length and width are in centimetres.

In this relation *area* is the dependent variable (it depends on the values of *length* and *width*) and *length* and *width* are two independent variables.

You don't need to worry about how to express these relations as tables or graphs (though just for fun you might like to think about how it could be done). Just the formula form will do us here.

You will probably realise that if we want to find the area of a rectangle, we have to know both the length and the width. So, to find the dependent variable, area, we substitute for both independent variables and solve the resulting equation.

Suppose the rectangle is 10 cm long and 4 cm wide. We substitute these values for *length* and *width* like so:

Formula:	*area* = *length* × *width*
Substituting gives :	= 10 × 4
And this gives:	= 40

Not too difficult eh? If there are three or more independent variables, just substitute for each one and calculate.

Practice

Q16 Use the formula *area* = *length* × *width* to find the areas of the following rectangles:

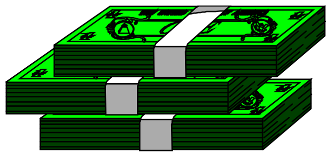
(a) length = 10 cm width = 6 cm

(b) length = 100 cm width = 5.2 cm

Q17 The formula for the simple interest paid on an investment is

interest = *principal* × *rate* × *time* ÷ 100.

(*Principal* is the amount invested, *rate* is the interest rate in percent per year and *time* is the number of years the money is invested for.)



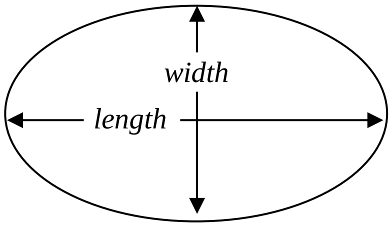
Find the interest paid on the following investments

(a) *principal* = $1000, *rate* = 5%, *time* = 2 years

(d) *principal* = $10 000, *rate* = 4%, *time* = 3½ years

Q18 The area of an ellipse is given by the formula

area = 0.785 × *length* × *width*.



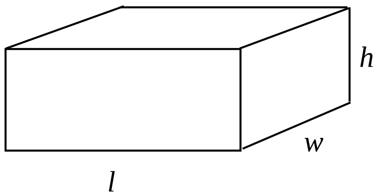
Find the areas of the following ellipses:

(a) 20 cm long, 10 cm wide

(b) 1 m long, 50 cm wide

Q19 The formula for the area of the surface of a rectangular prism is

area = 2 × (*length* × *width* + *width* × *height* + *height* × *length*).



Find the surface areas of the following rectangular prisms:

(a) length = 10, width = 5, height = 3

(b) 19.2 cm by 11.5 cm by 6.1 cm

Note that to avoid errors in this question, it is worth starting your working by writing down the values of *length*, *width* and *height*, e.g. *length* = 50 etc., then writing the formula, then substituting. Don't forget that, when calculating, you have to work out the bit in brackets first, then multiply the result by 2.

Q20 *rister* = (*artis* × 3 + *beta* × 2) ÷ (*sloff* + 1).

Find the value of *rister* if

(a) *artis* = 2, *beta* = 5 and *sloff* = 3

(b) *artis* = 5, *beta* = 2 and *sloff* = 0.5

Solve

Q51 During exercise, the maximum recommended pulse rate in beats per minute is given by the formula: *pulse rate* = 0.8 × (220 – *age*)
How much lower is the maximum recommended rate for a 34 year old than for an 18 year old?

Q52 For an adult 170 cm tall, BMI (body mass index) is given by their mass (in kg) divided by 2.89: *bmi* = *mass* ÷ 2.89.

Decide on a suitable domain for this relation, then present it as a graph.

Q53 The BMI for a person of any height is given by the formula *bmi* = *mass* ÷ $height^2$, where *mass* is in kg and *height* is in metres. Find the BMI for a person who is 1.8 m tall and who weighs 87 kg.

Q54 Using a process of guess and check or another method, convert the following relation to a formula:

Distance (km)	0	1	2	3	4	5
Taxi fare ($)	3	5	7	9	11	13

Revise

Revision Set 1

Q61 Mawler is a bouncer. He charges for jobs according to the time spent. He uses the following formula:
charge = 50 + 40 × *time*, where *charge* is in dollars and *time* is in hours.
How much would he charge for a job that took 3½ hours?

Q62 Use the formula *mass* = 60 + *age* × 2 to find the mass if the age is 40.

Q63 It has been discovered that the number of passengers carried on a particular route is related to the number of buses that run per day. The number of buses can vary from 6 to 12. The relation can be expressed by the formula

number of passengers = 70 + *number of buses* × 8

Express this relation as a set of ordered pairs, as a table and as a graph.

Q64 The mass in kg of a can with petrol in is related to the volume in litres of petrol by the relation

mass = 1.2 + *volume* × 0.8

The can has a capacity of 5 litres and this determines the domain.

Present this relation as a table and as a graph.

Q65 The formula for the simple interest paid on an investment is

interest = *principal* (in dollars) × *rate* (in %) × *time* (in years) ÷ 100.
Find the interest paid if: *principal* = \$500, *rate* = 12%, *time* = 3 years

Q66 *distance travelled* = *initial speed* × *time* + ½ × *acceleration* × $time^2$
Find the distance travelled if the acceleration is 4, the time is 10 and the initial speed is 20.

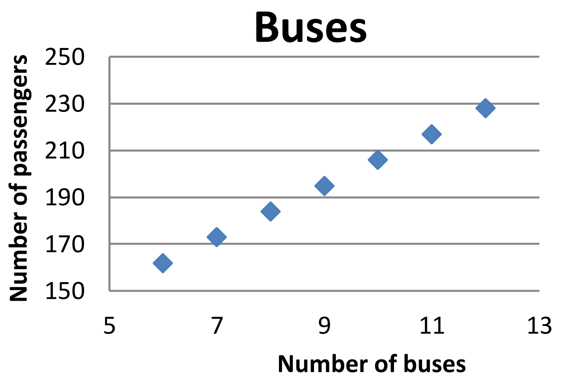
Answers

Q1. (a) 22 (b) 181
Q2. (a) 16 (b) 157
Q3. (a) 13 (b) 1555
Q4. (a) 21 (b) 381
Q5. (a) 30 cm (b) 2.1 m
Q6. (a) \$13 (b) \$41.40
Q7. (a) \$55 (b) \$32.50
Q8. (a) 7.5 (b) 151
Q9. (a) 994 (b) 250
Q10. Set of ordered pairs: (6, 162), (7, 173), (8, 184), (9, 195), (10, 206), (11, 217), (12, 228), where the first number if the number of buses that run and the second number is the number of passengers taken.

Table

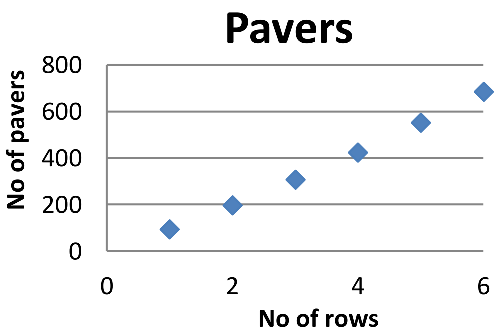
Number of buses	6	7	8	9	10	11	12
Number of passengers	162	173	184	195	206	217	228

Graph

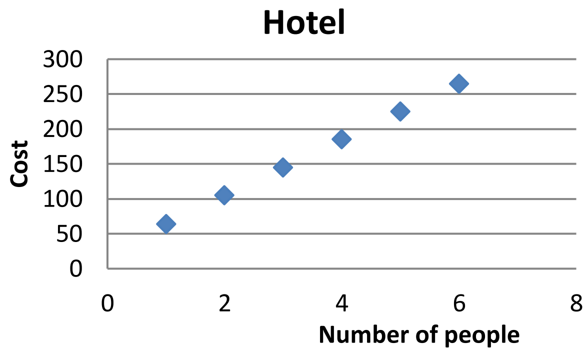


Q11.

Number of rows	1	2	3	4	5	6
Number of pavers	94	196	306	424	550	684

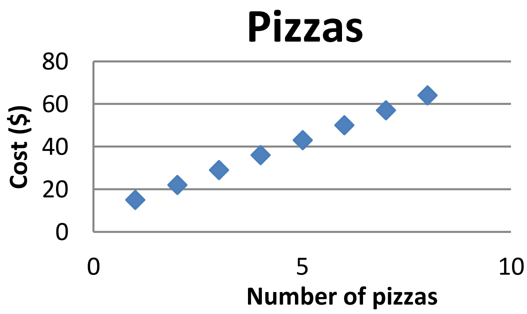


Q12. The domain is the whole numbers from 1-6 inclusive.
(1, 65), (2, 105), (3, 145), (4, 185), (5, 225), (6, 265), where the first number is the number of people and the second number is the cost in dollars.



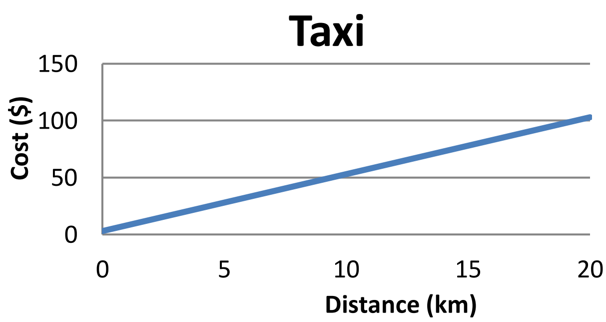
Q13.

Number of pizzas	1	2	3	4	5	6	7	8
Cost ($)	15	22	29	36	43	50	57	64



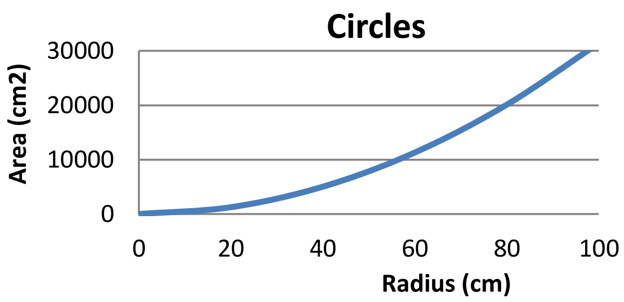
Q14.

Distance (km)	0	2	4	6	8	10	12	14	16	18	20
Cost ($)	3	13	23	33	43	53	63	73	83	93	103



Q15. (0, 0), (20, 1256), (40, 5024), (60, 11 304), (80, 20 096), (100, 31 400), where the first number is the radius in cm and the second is the area in cm^2.

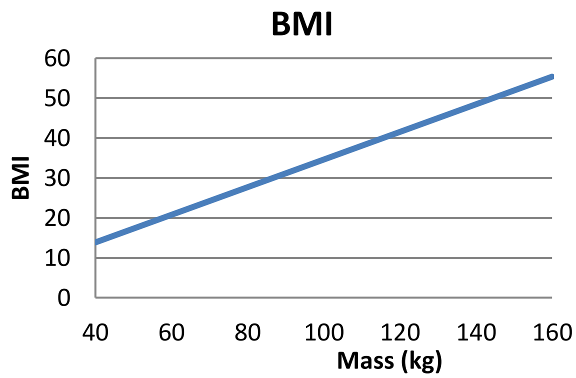
Radius	0	20	40	60	80	100
Area	0	1256	5024	11 304	20 096	31 400



Q16. (a) 60 cm² (b) 520 cm²
Q17. (a) $100 (b) $1400
Q18. (a) 157 cm² (b) 0.3925 m²
Q19. (a) 190 (b) 816 cm²
Q20. (a) 4 (b) 12.67

Q51. (a) 12.8 bpm

Q52.



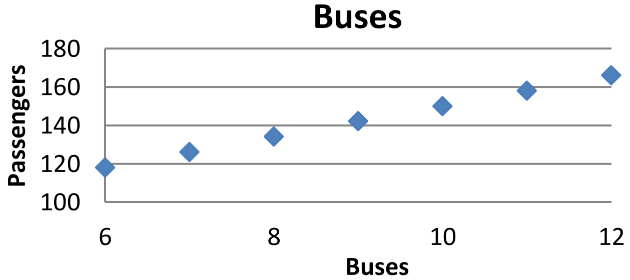
Q53. 26.9
Q54. *fare* = 3 + *distance* × 2

Q61. $190
Q62. 140
Q63. Set of ordered pairs: (6, 118), (7, 126), (8, 134), (9, 142), (10, 150), (11, 158), (12, 166), where the first number if the number of buses that run and the second number is the number of passengers taken.

Table

Number of buses	6	7	8	9	10	11	12
Number of passengers	118	126	134	142	150	158	166

Graph

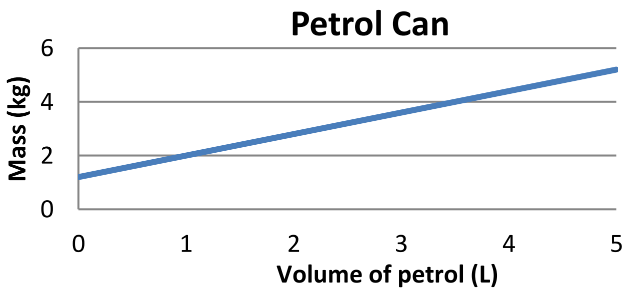


Q64.

Table

Volume or petrol	0	1	2	3	4	5
Mass (kg)	1.2	2.0	2.8	3.6	4.4	5.2

Graph



Q65. $180 Q76. 400