M1 Maths

A1-3 Patterns

- recognise patterns in sequences of numbers
- determine whether a relation has a pattern (from both tables and graphs) and know that relations with patterns can be expressed as formulae

Summary Learn Solve Revise Answers

Summary

Sequences of numbers can have patterns which may or may not be obvious at first sight. A good way to find patterns that aren't obvious is the difference method.

A relation has a pattern if there is a pattern in the values for the dependent variable when the value pairs are arranged so that there is a pattern in the values for the independent variable.

Relations with patterns can be expressed as formulae.

Learn

Patterns in Sequences of Numbers

A **sequence** is a set of things in a particular order. Red, yellow black is a sequence of colours. Yellow, black, red is the same set, but it's a different sequence because the colours are not in the same order.

25, 29, 14, 20, 17, 9, 22, 17, 41 is a sequence of numbers.

17, 19, 21, 23, 25, 27, 27, 31 is another sequence. This sequence differs from the first sequence in that it has an obvious **pattern**: each number is two more than the previous number.

Because the second sequence has a pattern, it is possible to predict how it would continue: it will keep going up in 2s: 33, 35, 37, etc. It isn't possible to tell how the first sequence will continue. We say that a sequence has a pattern if we can tell how it will continue.

Identifying Patterns

The pattern above is easy to spot. These ones might be a little harder. See how many you can spot. For the ones you can, write down the next number. Do this before you read on.

(a) 1, 11, 111, 1111, 11111, …

(b) 115, 108, 101, 94, 87, 80, ...

(c) 4, 7, 8, 6, 4, 7, 8, 6, 4, 7, …

(d) 5, 10, 20, 40, 80, …

(e) 5, 7, 12, 19, 31, 50, 81, …

(f) 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, …

This is what you should have found.

(a) the number of 1s keeps increasing by one; the next number is 111111.

(b) each number is 7 less than the previous number; 73.

(c) the sequence 4-7-8-6 keeps repeating; 8.

(d) the numbers keep doubling; 160.

(e) each number is the sum of the previous two; 131.

(f) these are the square numbers; 121

The Method of Differences

Other patterns can be less obvious, like the ones in the three sequences below.

(g) 22, 25, 30, 37, 46, 57, ...

(h) 13, 25, 46, 75, 111, 153, 200, 251, ...

(i) 11, 19, 43, 115, 331, 979, ...

For these we can use the **method of differences**. We look at the differences between successive numbers in the sequences.

Sequence (g) can be dealt with like this:

Sequence: 22 25 30 37 46 57

Differences: 3 5 7 9 11

These differences do have a more obvious pattern and can be continued like this:

22 25 30 37 46 57

3 5 7 9 11 13 15 17

Once this is done, the original sequence can be continued by adding on the differences:

22 25 30 37 46 57 70 85 102

3 5 7 9 11 13 15 17

Sequence (h) can be approached in the same way.

13 25 46 75 111 153 200 251

12 21 29 36 42 47 51

There is still no obvious pattern. So we can go on to find the second differences – the differences between the differences – like this:

13 25 46 75 111 153 200 251

12 21 29 36 42 47 51

9 8 7 6 5 4

These second differences do have an obvious pattern, so we can continue like this.

13 25 46 75 111 153 200 251 305 361

12 21 29 36 42 47 51 54 56

9 8 7 6 5 4 3 2

We could even go on to the third differences and beyond, but you won't be expected to do that.

Sequence (i) gives us:

11 19 43 115 331 979

8 24 72 216 648

16 48 144 432

It will never settle down to a simpler pattern, but, instead, will keep producing sequences where each number is 3 times the previous one. The next number is 2923.

So when you need to identify a pattern in a sequence of numbers, look for something obvious and, if that doesn't work, try looking at differences. This will work for most patterns that you will meet, though not all. It won't work for

1, 10, 11, 100, 101, 110, 111, 1000, 1001, ...,

Practice

Q1 For each of the following sequences, try to find the pattern (by looking at it and, if that doesn't work, by using the method of differences). Then describe the pattern (for instance you might say that the second differences go up in 2s). Then work out the next number.

(a) 71, 75, 79, 83, 87, 91, ...

(b) 110, 97, 84, 71, 58, ...

(c) 27, 54, 108, 216, 432, ...

(d) 6, 8, 14, 32, 86, 248, ...

(e) 29, 31, 38, 50, 67, 89, 116, ...

(f) 4, 12, 4, 12, 4, 12, ...

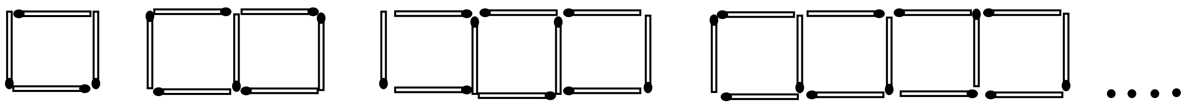
(g) 1, 1, 1, 3, 5, 9, 17, 31, 57, ...

(h) 2, 9, 28, 65, 126, 217, ...

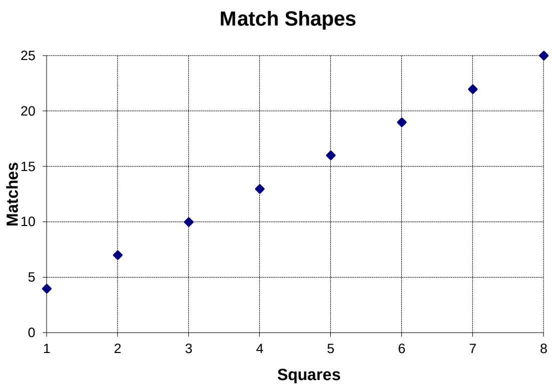
Patterns in Relations

Examine the following two relations. Each is expressed as a table and as a graph.

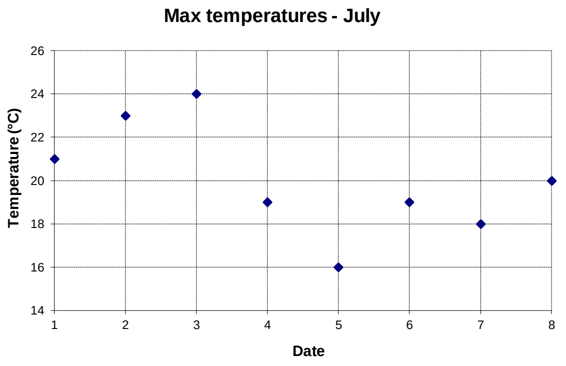
Relation 1: Numbers of matches required to make various length rows of squares



Squares	1	2	3	4	5	6	7	8
Matches	4	7	10	13	16	19	22	25



Relation 2: Maximum temperatures for the first eight days in July



Day	1	2	3	4	5	6	7	8
Temp	21	23	24	19	16	19	18	20

From the information presented, try to answer the following questions:

1. Use Relation 1 to predict the number of matches needed to make a row of 9 squares;
2. Use Relation 2 to predict the temperature for 9 July.

Do this before you go on.

How did you go with Question 1? The answer is 28 matches. There is a definite pattern in the data in Relation 1. It is quite easy to predict the numbers of matches for 9, 10, 11 etc. squares. The numbers just keep going up by 3 for each extra square.

In general a relation has a pattern if, when the independent variable values form a sequence with a pattern, then the dependent variable values also form a sequence with a pattern.

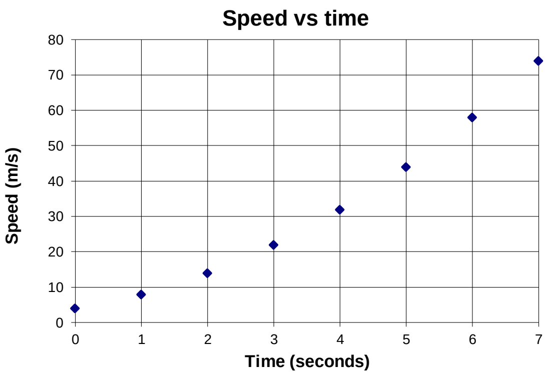
Question 2, however, is a different matter. There is no definite pattern in the data and so it is not possible to predict the temperature on subsequent days, except by educated guesswork. 21° would be as good a guess as 17° or, indeed, any other temperature in that general range.

This illustrates a very important aspect of relations:

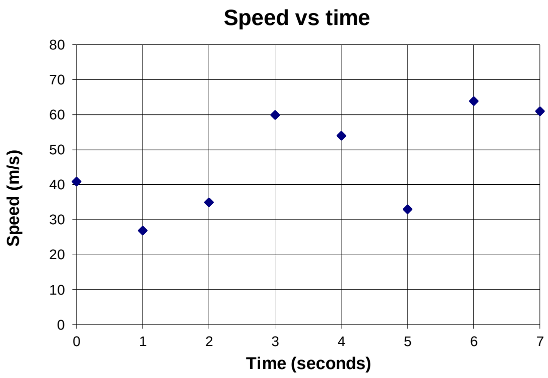
Some relations have a definite pattern and some don't.

If the relation is given as a graph, then, if the points on the graph form a particular shape or pattern and you can see how that shape or pattern would continue, then the relation has a pattern. If the points seem somewhat randomly scattered, then the relation doesn't have a pattern.

Pattern



No pattern



Practice

Q2 For each of the following relations, say whether it has a definite pattern and if so, find the next two pairs of values.

(a)

Quantity 1	1	2	3	4	5	6	7	8
Quantity 2	14	12	10	8	6	4	2	0

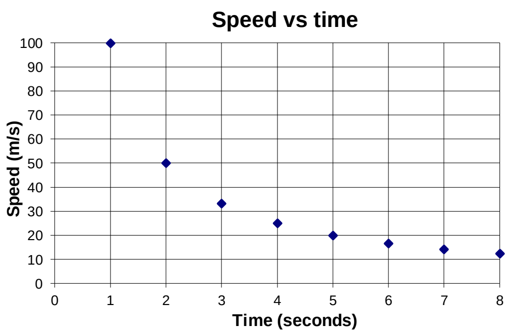
(b)

Variable 1	4	6	8	10	12	14	16	18
Variable 2	14	16	18	30	32	35	38	36

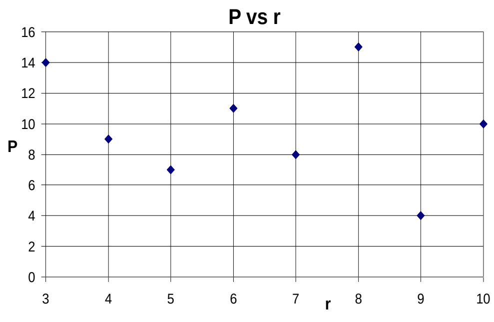
(c)

Variable 1	3	4	5	6	7	8
Variable 2	6	12	24	48	96	192

(d)

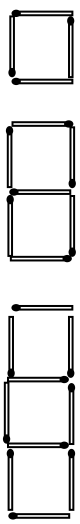


(e)



Formulae

When a relation has a pattern, we can describe the pattern rather than list all the pairs of values. This is often much quicker. For example, if we were interested in the number of matches needed to make numbers of squares from 1 to 50, we could write the relation as follows:



Squares	1	2	3	4	5	6	7	8	9	10
Matches	4	7	10	13	16	19	22	25	28	31
Squares	11	12	13	14	15	16	17	18	19	20
Matches	34	37	40	43	46	49	52	55	58	61
Squares	21	22	23	24	25	26	27	28	29	30
Matches	64	67	70	73	76	79	82	85	88	91
Squares	31	32	33	34	35	36	37	38	39	40
Matches	94	97	100	103	106	109	112	115	118	121
Squares	41	42	43	44	45	46	47	48	49	50
Matches	124	127	130	133	136	139	142	145	148	151

or we could describe the pattern by saying:

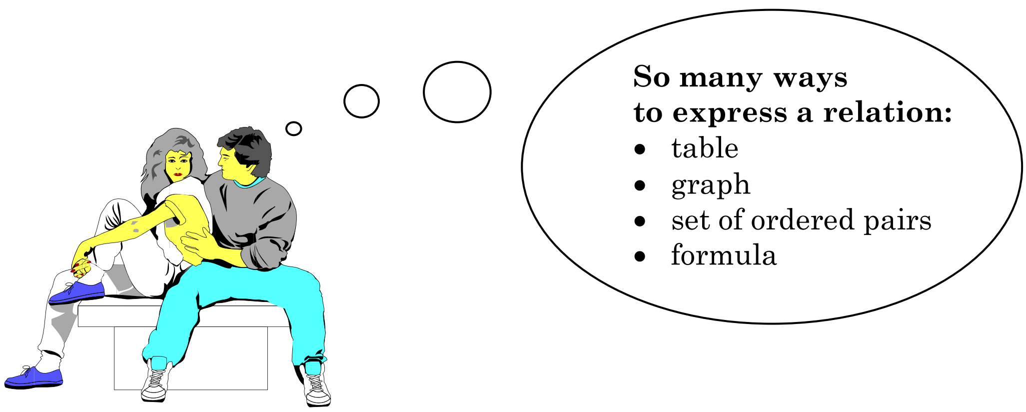
***the number of matches* is 3 times *the number of squares* plus 1.**

This can be written even shorter like this:

***number of matches* = *number of* squares × 3 + 1**

Such a description of a relation is called a **formula**. The plural of formula can be either formulas or formulae (pronounced "formulee").

Just like a set of ordered pairs, a table or a graph, a formula allows us to find the value of one variable if we know the value of the other. But we will learn how to do this in later modules.



Here are some other examples of formulae.

An electrician might work out how much to charge by using the formula

***charge* = *number of hours* × $50 + $35**.

For a circle

***circumference* = *diameter* × 3.14**

For a taxi with a $2.50 flag fall and $2 per kilometre

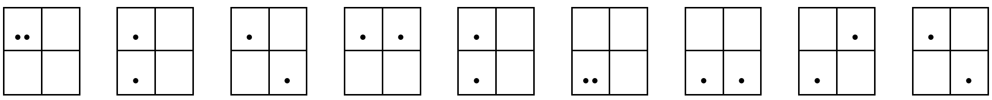
***fare* = $2.50 + *distance* × $2**

Note that in a formula, the dependent variable is generally by itself on the left side of the = sign; the independent variable and some numbers are generally on the right side. So in the fare formula above, *fare* is the dependent variable and *distance* is the independent variable.

Solve

Q51 Here are some sequences of things other than normal numbers. They all have patterns. For each one, see if you can spot the pattern, then explain it and give the next two members of the sequence. Some are quite tricky.

(a) May, October, March, August, January, June, ..., ...

(b)  ..., ...,

(c) A, B, D, G, K, P, V, C, K, T, D, O, A, N, ..., ...

(d) A, B, D, H, P, F, L, X, V, ..., ...

(e) 1, 10, 11, 100, 101, 110, 111, 1000, 1001, ..., ...

(f)  ..., ...

(g) 1, 11, 21, 1211, 111221, 312211, 13112221, 1113213211, 31131211131221, 13211311123113112211, ..., ...

Q52 (a) Find the 20^{th} number in this sequence: 2, 4, 6, 8, 10, 12, 14, ...

(b) Find the 1000^{th} number in: 2, 4, 6, 8, 10, 12, 14, ...

(c) Find the 20^{th} number in: 13, 15, 17, 19, 21, 23, ...

(d) Find the 2000^{th} number in: 34, 38, 42, 46, 50, 54, ...

(e) Find the 20^{th} number in: 1, 2, 4, 8, 16, 32, ...

(f) If today is Friday, what day will it be 7001 days from today?

(g) If today is Monday, what day will it be 15 823 days from today?

Q53 (a) $3^1=3=3$, $3^2=3\times3=9$, $3^3=3\times3\times3=27$, $3^4=3\times3\times3\times3=81$, $3^5=3\times3\times3\times3\times3=243$, $3^6=3\times3\times3\times3\times3\times3=729$, $3^7=3\times3\times3\times3\times3\times3\times3=2187$.

Find the last digit of 3^{60}, i.e.
3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3
×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3×3.

(b) Find the last digit of 7^{159}.

Revise

Revision Set 1

Q61 For each of the following relations, say whether it has a pattern and, if it does, find the next two ordered pairs.

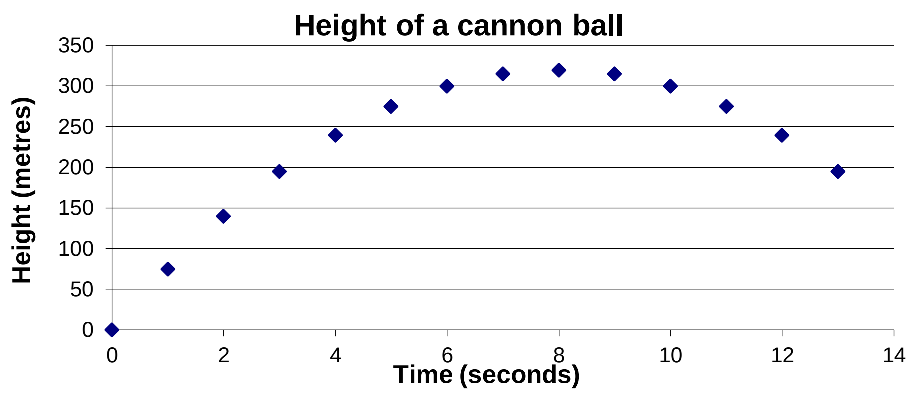
(a)

Age	0	10	20	30	40	50	60	70
Mass (kg)	3	13	22	30	37	43	48	52

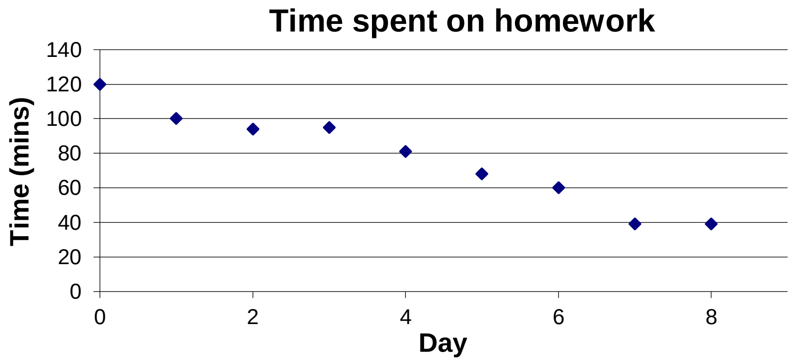
(b)

Number ordered	1	2	3	4	5	6	7
Price	5	6.5	8	9.5	11	12.5	14

(c)



(d)



Q62 How can a relation with a pattern be expressed other than as a set of statements, set of ordered pairs, table and graph?

Q63 Give an example of a formula.

Answers

Q1 (a) increasing in 4s, 95 (b) decreasing in 13s, 45 (c) doubling, 864
(d) adding 2, 6, 18, 54, each 3 times the last, 734
(e) increasing 12, 7, 12, 17, 12, 27 . . ., 118 (f) repeating the same two numbers, 4
(g) each number is the sum of the previous three, 105
(h) third differences are all 6, 344

Q2 (a) yes, (9, -2), (10, -4) (b) no (c) yes, (9, 384), (10, 768)
(d) yes, (9, 11), (10, 10) roughly (e) no

Q51 (a) steps of 5 months, November, April
(b) the dots move around anti-clockwise, one moves one position every step, the other every four steps, the next will have dots bottom left and bottom right, the one after both bottom right
(c) advancing through the alphabet 1 letter, 2 letters, 3 letters and so on, B, Q
(d) position in alphabet doubling, alphabet repeating, R, J
(e) counting in binary, 1010, 1011
(f) the numerals 1, 2, 3 etc. reflected in a line down their left side, 88 , pq
(g) each term is a description of the digits in the previous term, e.g. the 5^{th} term says one 1, one 2, two 1s, 11131221133112132113212221, 3113112221232112111312211312113211

Q52 (a) 40 (b) 2000 (c) 51 (d) 4030 (e) 524288 (f) Saturday
(g) Thursday

Q53 (a) 1 (b) 3

Q61 (a) yes, (80, 55), (90, 57) (b) yes, (8, 15.5), (9, 17) (c) yes, (14, 140), (15, 75) (d) no

Q62 as a formula S3. *fare* = *distance* × 2 + 3, where fare is in dollars and distance is in kilometres