M1 Maths

A1-1 Relations 1

- use relations given as tables, as graphs and as sets of ordered pairs

Summary Learn Solve Revise Answers

Summary

A relation is information which allows us to find the value of one quantity if we know the value of another quantity.

It consists of a set of value pairs.

It can be expressed as a table, a graph or a set of ordered pairs.

Learn

What is a Relation?

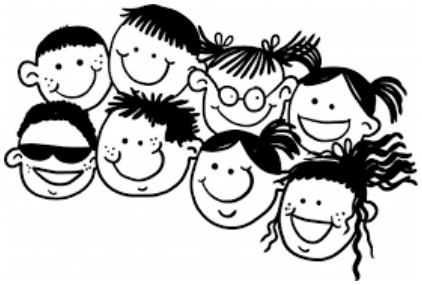
Families can join the Matrix Adventure Club. The cost of joining depends on the number of children in the family.

The costs are as follows:

Number of Children	Cost ($)
1	20
2	35
3	50
4	60
5	70
6	70

The information in this table is a **relation** – the relation between the number of children and the joining cost. The relation allows us to find the cost if we know the number of children or to find the number of children if we know the cost.

For instance, we can read off that, if there are four children, then the cost is $60. We can also read off that, if the cost is $35, then the number of children is 2.



Quantities and Values

There are two **quantities** involved in this relation

- one is the number of children in the family;
- the other is the cost or the amount they have to pay.

Each quantity can have a few different **values**. The quantity *number of children* has values 1, 2, 3, 4, 5 and 6 in this relation. The quantity *cost* has values \$20, \$35, \$50, \$60, and \$70. A value is a particular number that a quantity can take.

The relation consists of a set of pairs of values or **value pairs**. A value pair is a value for one quantity along with the corresponding value for the other quantity. In the relation above, there are 6 value pairs. Each row in the table is one value pair – a value for the quantity *number of children* and a corresponding value for the other quantity *cost*.

A relation is information which allows us to find the value of one quantity if we know the value of another quantity.

It consists of a set of value pairs.

Learn this definition, then make sure you can write it down without looking back at it. If you don't get it right, then continue to learn it and try again.

Ways of Expressing a Relation

A relation can be written as a **table** like the one above, but it can also be written as a **graph** or as a **set of ordered pairs**. You will now learn how to write and use relations in each of these ways.

Relations as Tables

The Matrix relation between number of children and joining cost can be expressed as a table like this:

Children	Payment ($)
1	20
2	35
3	50
4	60
5	70
6	70

or round the other way like this:

Children	1	2	3	4	5	6
Payment ($)	20	35	50	60	70	70

In the first table, each horizontal row is a value pair; in the second table, each vertical column is a value pair.

To use a table to find the joining cost for a given number of children, locate the number of children in the left column (or top row) and read across to the right column (or down to the bottom row). To find the number of children given the joining cost, locate the joining cost in the right column (or bottom row) and read back to the left column (or top row). Make sure you can do this with the tables above.

Practice

Q1 Use one of the tables above to find the joining cost for a family with 5 children.

Q2 Use one of the tables above to find the number of children in a family that pays $50 to join.

Q3 The following table shows the relation between standard men's shoe size and the length of the shoe in centimetres.

Shoe size	4	5	6	7	8	9
Length (cm)	26.0	26.8	27.6	28.3	29.0	29.8

(a) How long is a size 4 shoe?

(b) How long is a size 8 shoe?

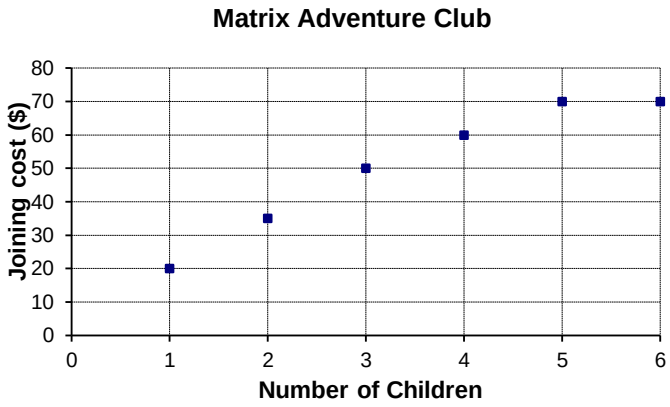
(c) If you measured a shoe and found it to be 28.3 cm long, what size would it be?

(d) Another shoe is 27.4 cm long. What size is it most likely to be?

(e) On his 12th birthday, Harry wore a size 4 shoe. On his 14th birthday he wore a size 7 shoe. How many centimetres did his foot grow in those two years?

Relations as Graphs

The relation between number of children and joining cost can be expressed in the form of a graph in various ways. The example below is the simplest.



In this graph, each data point represents one value pair from the relation. The third point (above the 3) is level with the 50 on the cost axis and so shows that, if the number of children is 3, then the joining cost is $50. Make sure you can see this before you go on. Of course the graph can be read the other way. If the joining cost is $50, then the number of children is 3.

Graphs are useful in that trends in the data can be seen at a glance. However, they often cannot be read as accurately as a table. For example, from the graph above, it is possible only to estimate the payment for 2 children at about $35. If it were $34.80, we probably wouldn't notice the difference and we would still read the graph as $35.

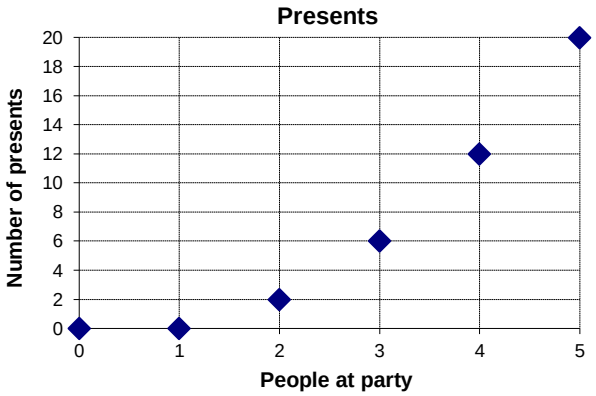
Note that, in this graph, the points are not joined with lines. Doing so would suggest that the graph has meaning at any point along the lines. One might then conclude that a family with 3.5 children would pay $55. This is not implied by the relation. A family with 3.5 children would more likely be paid a visit by the homicide squad.

Practice

Q4 Use the Matrix graph above to find the joining cost for a family with 5 children.

Q5 (a) Use the Matrix graph above to find the number of children if the family has to pay $20.

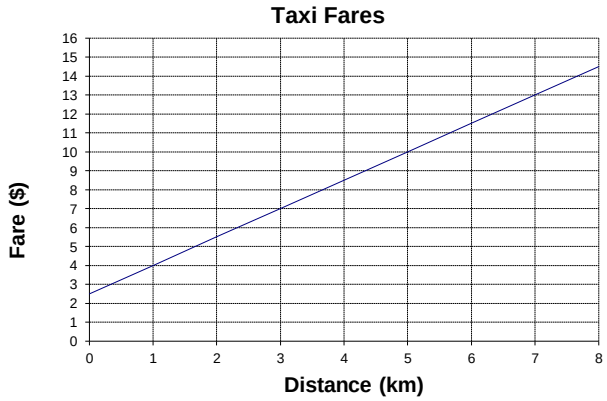
Q6 A number of people go to a party and everyone takes everyone else a present. The relation between the number of people and the number of presents is given in the graph below.



(a) How many presents are taken if 4 people go?

(b) How many people went if there were 6 presents?

Q7 (a) Use the taxi fare graph below to find the fare for a trip of 6 km.



(b) Use the taxi fare graph to find how far you can go by taxi for $5.50.

(c) When you get in a taxi, you are charged a 'flag fall', that is an amount of money even if you don't go anywhere. Then you are charged an amount per kilometre travelled on top of that. Use the graph to find the flag fall.

(d) How much do you pay per kilometre after the flag fall?

Note that the graph in the last question is a line rather than a set of separate points. This is because there is a fare for 3.5 km or 3.7164 km, indeed for any distance.

Relations as Sets of Ordered Pairs

The third way of expressing a relation is as a set of ordered pairs. In a set of ordered pairs we take each pair of values and put them in round brackets with the two values separated by a comma. The Matrix relation would look like this:

(1, 20), (2, 35), (3, 50), (4, 60), (5, 70), (6, 70)
where the first number is the number of children, the second number is the cost in dollars

The pairs are ordered in the sense that the two numbers in each pair always have to be in the same order – in this case, number of children first, cost second. This is so we know which number is which when we read them. Also, there has to be a statement indicating what the first and second numbers in each pair are. This is the second line in the relation above.

Another example of a relation expressed as a set of ordered pairs is the following: (0, 3), (1, 9), (2, 13), (3, 16), (4, 20), (5, 22), (6, 25), (7, 27), where the first number is the age of a child in years and the second number is her mass in kilograms.

Note that, in the explanation of what the numbers mean, the units are stated (years for age and kilograms for mass). This is important.

Practice

Q8 The cost (including postage and handling) for 1 to 5 towels from a mail order club is as follows:

One towel costs $14.95
Two towels cost $24.90
Three towels cost $34.85
Four towels cost $44.80
Five towels cost $52.50

Write this relation as a set of ordered pairs. Don't forget to include a statement saying what the first and second numbers in each pair are.

Q9 The relation between cooking time and mass for legs of pork is as follows:

(4, 1.5), (5, 1.8), (6, 2.0), (7, 2.1), (8, 2.2) where the first number is the mass in kilograms and the second number is the cooking time in hours.

(a) For how long should a 4 kg leg be cooked?

(b) What size leg needs to be cooked for 2 hours?

(c) How much longer does a 7 kg leg take than a 5 kg leg?

Q10 Make up a few (at least 3) more relations concerning day-to-day situations. Express each as a table, as a graph and as a set of ordered pairs.

Relations as Sets of Statements

The information in a relation could be presented as a set of statements. For example, we could express the Matrix relation like this:

if the number of children is 1, the cost is $20;
if the number of children is 2, the cost is $35;
if the number of children is 3, the cost is $50;
if the number of children is 4, the cost is $60;
if the number of children is 5, the cost is $70;
if the number of children is 6, the cost is $70.

This is a cumbersome and inefficient way of writing it though and is not often used by mathematicians. You won't be asked to write a relation as a set of statements, although you might be given a relation as a set of statements, as you were in Q8 above. It should be fairly obvious what to do.

Solve

Q51 The following is the relation between the diameter of a sphere and its volume.

Diameter (cm)	0	5	10	15	20
Volume (cm^3)	0	65	523	1767	4189

Express this relation as an accurate graph. Because spheres with diameters between the given values have volumes, you can connect the points on the graph with a curve.

Use the graph to find, as accurately as you can, the volumes of spheres with the following diameters:
(a) 10 cm (b) 20 cm (c) 17 cm (d) 1 cm (e) 12.5 cm

Use the graph to find, as accurately as you can, the diameters of spheres with the following volumes:
(f) 1767 cm^3 (g) 500 cm^3 (h) 55 cm^3 (i) 1000 cm^3 (j) 12 cm^3

Q52 The following is the relation between surface area and diameter for a sphere.

Diameter (cm)	Surface area (cm^2)
0	0
5	79
10	314
15	707
20	1257

Use this relation along with the relation in S1 to write the relation between surface area and volume for a sphere. Write it as a set of ordered pairs with

volume as the first value in each pair, and as a graph with volume on the horizontal axis.

Q53 Redo Q52, but this time, make surface area the first value in each ordered pair and the quantity on the horizontal axis in the graph.

Revise

Revision Set 1

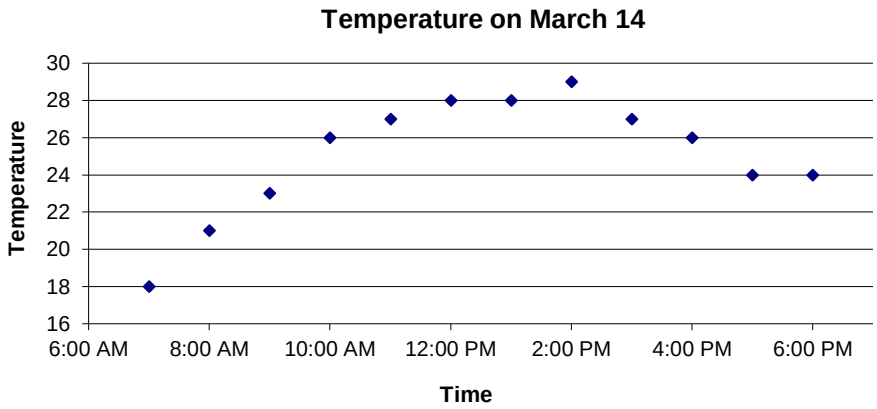
Q61 The relation below shows, as a table, average life expectancy for females of various ages in 17th Century England. Use it to find:

(a) the life expectancy of a 20-year old

(b) the age you have to reach before your life expectancy is 64

Age	0	20	40	60	80
Life expectancy	36	49	64	70	85

Q62 The relation between temperature and time for March 14 is shown below.



(a) What was the temperature at 9 a.m.?

(b) When did the temperature first reach 27°?

Q63 The relation below is given as a set of ordered pairs. The first number is the age of a child in years, the second number is their heart rate in beats per minute.

(0, 125), (1, 108), (2, 102), (3, 94), (4, 94), (5, 89), (6, 90)

Use it to find:

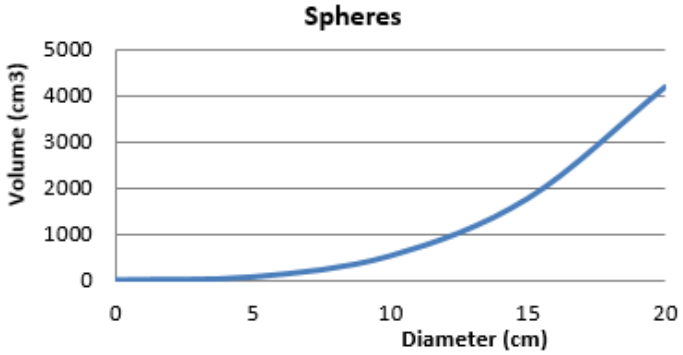
(a) the heart rate when the child was 4

(b) the age at which the heart rate was 108

Answers

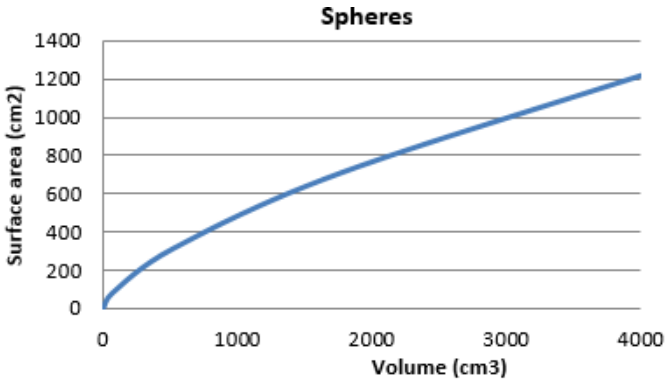
Q1 $70 Q2 3 Q3 (a) 26 cm (b) 29 cm (c) 7 (d) 6 (e) 2.3
Q4 $70 Q5 1 Q6 (a) 12 (b) 3
Q7 (a) $11.50 (b) 2 km (c) $2.50 (d) $1.50
Q8 (1, 14.95), (2, 24.90), (3, 34.85), (4, 44.80), (5, 52.50), where the first number is the number of towels bought, the second number is the cost in dollars
Q9 (a) 1.5 h (b) 6 kg (c) 0.3 h Q10 -

Q51

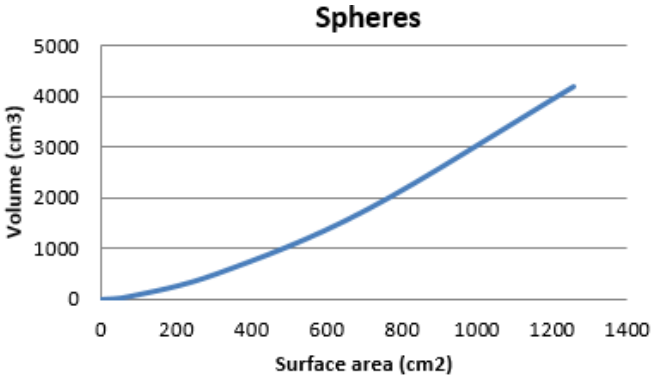


(a) 520 cm^3 (b) 4190 cm^3 (c) 2570 cm^3 (d) 1 cm^3 (e) 1020 cm^3
(f) 23.1 cm (g) 15.2 cm (h) 7.3 cm (i) 19.1 cm (j) 4.4 cm

Q52 The first number is the volume in cm^3, the second is the surface area in cm^2:
(0, 0), (65, 79), (523, 314), (1767, 707), (4189, 1257)



Q53 The first number is the volume in cm^3, the second is the surface area in cm^2:
(0, 0), (79, 65), (314, 523), (707, 1767), (1257, 4189)



Q61 (a) 49 years (b) 40 Q62 (a) 23° (b) 11 a.m. Q63 (a) 94 (b) 1